FEATURES OF DELOCALIZATION OF PARTICLES FROM A RECTANGULAR POTENTIAL WELL AT DECREASE OF ITS DEPTH WITH TIME

A.Ch. IZMAILOV
Institute of Physics, Azerbaijan Republic Ministry of Sciences and Education, Baku, AZ-1143, Azerbaijan
e-mail: azizm57@rambler.ru

The process of delocalization of particles from the one-dimensional rectangular potential well with a decrease in its depth with time is theoretically investigated. In practice, such a well corresponds, in particular, to a region of a local quasi-homogeneous electric (or magnetic) field created along the direction of particle motion with a dipole electric (or magnetic) moment. It is believed that these particles are in a high vacuum and the electromagnetic forces acting on them are not dissipative. The conditions under which a multiple slowdown (cooling) of delocalized particles is achieved in comparison with their initial state in the potential well are established and analyzed.

Keywords: delocalization and slowdown of particles, rectangular potential well, high vacuum.
PACS: 45.50.-j, 41.20.-q, 42.62.Fi

1. INTRODUCTION

The development of effective methods for slowdown (cooling) of various micro- and nanoparticles in vacuum is important for studying the fundamental problems of gravity and quantum mechanics using these particles, including limits of applicability of the corresponding theories [1, 2]. Moreover, deep cooling of atoms and molecules in vacuum is necessary for precision fundamental research [3].

The author's articles [4, 5] showed the possibility of slowdown and trapping of various particles (including atoms and molecules in their ground quantum state) by means of external electromagnetic fields, which induce potential wells on a path of such particles with a fixed spatial distribution, but deepening with time up to some limit. It is assumed that the considered particles are in high vacuum conditions and forces acting on them are not dissipative, i.e. these particles move without friction. Depending on whether the particles have an electric (magnetic) dipole moment, a controlled electric (magnetic) field can be used to capture or slowdown them by the proposed methods [4, 5].

In the brief communication [6], author showed the possibility of significant slowdown (cooling) of such particles in high vacuum due to their removal from the initial localized state in a similar potential well with a sufficiently slow decrease in the depth of this well by means of a controlled weakening of the corresponding electromagnetic field.

In the general case, for study of such processes, it is necessary to numerically solve the differential equations of particle motion [6]. However, a number of important features of such a deceleration (cooling) of particles will be established in this work on the basis of simple mathematical relations for the one-dimensional rectangular potential well. In practice, such a well corresponds, in particular, to a region of a local quasi-homogeneous electric (or magnetic) field created along the direction of particle flight with a dipole electric (or magnetic) moment.

2. BASIC RELATIONSHIPS

As in articles [4-6], we will conduct theoretical study within the framework of classical mechanics and electrodynamics. Let us assume that a point particle with mass m moves in vacuum, including the region of the potential well $U(\mathbf{r},t)$, which explicitly depends not only on the coordinate $\mathbf{r}$, but also on the time t. The total energy of the particle with a nonrelativistic velocity v is described by the well-known formula [7]:

$$E(\mathrm{r},\mathrm{v},t) = 0.5mv^2 + U(\mathrm{r},t). \quad (1)$$

We will consider the following potential energy $U(\mathrm{r},t)$:

$$U(\mathrm{r},t) = s(\mathrm{r}) * \varphi(t), \quad (2)$$

where the coordinate function $s(\mathbf{r}) \leq 0$ and $\varphi(t)$ is a function of time t. Such a non-stationary potential (2) can be created for particles with an electric or magnetic moment by means of a controlled electromagnetic field (in particular, non-resonant laser radiation) with a time-varying intensity, but with a fixed spatial distribution [4-6]. In this case (2), we obtain the following equation of particle motion [7]:

$$m\frac{d^2\mathbf{r}}{dt^2} = -\varphi(t)\frac{ds(\mathbf{r})}{d\mathbf{r}}. \quad (3)$$

We consider the situation when the force on the right side of equation (3) does not have a dissipative effect on the motion of the particle, i.e. is not a force of friction. Then from relations (1)-(3), the important formula follows for the time derivative of the total particle energy E:

$$\frac{dE}{dt} = s(\mathbf{r})\frac{d\varphi(t)}{dt}. \quad (4)$$

Further we will consider the one-dimensional rectangular potential well $U(x,t)$, bounded by the coordinates $x = \pm L$, whose depth decreases with time t (Fig. 1):

131, H.Javid ave, AZ-1143, Baku
Institute of Physics
E-mail: jophphysics@gmail.com

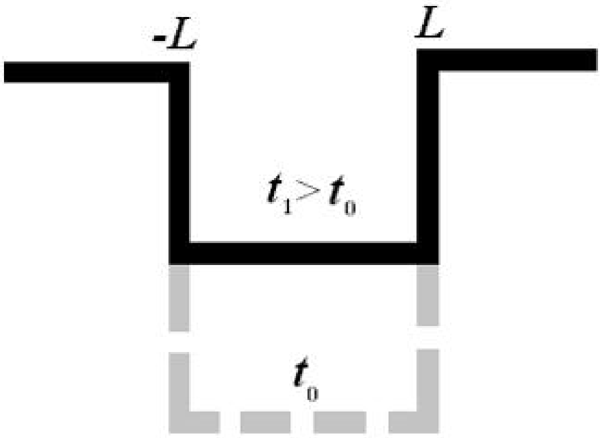

Fig.1. One-dimensional rectangular potential well with the depth decreasing during time t.

$$U(x,t) = -J_0 \eta(L^2 - x^2)\varphi(t), \quad (5)$$

where $J_0 > 0$ is the constant with the dimension of energy, $\eta(L^2 - x^2)$ is the step function ($\eta(y) = 1$ for $y \geq 0$ and $\eta(y) = 0$ for $y < 0$), $0 \leq \varphi(t) \leq 1$ is a non-increasing function of time. It is believed that until the moment $t = 0$ the potential well (5) is stationary with the value $\varphi(0) = 1$ and a point particle with mass m and velocity v_0 is localized in it. Then, taking into account (5), the initial total energy E_0 of this particle at $t = 0$ has the form:

$$E_0 = 0.5 m v_0^2 - J_0. \quad (6)$$

Localization of the particle in the well (5) is possible only at the energy $E_0 \leq 0$ (6), i.e. under the following limitation on the particle velocity v_0:

$$|v_0| \leq v_{\mathrm{m}} = \sqrt{\frac{2J_0}{m}}. \quad (7)$$

According to the equation of motion (3), the module $|v_0|$ (7) of the particle velocity will be constant within the one-dimensional rectangular potential well (5) despite the change in its depth. Starting from the moment $t = 0$, the depth of the well (5) starts to decrease. Such a process is accompanied by an increase in the total particle energy $E(t)$, according to relation (4), which in the case of a rectangular well (5) reduces to the following formula for $t \geq 0$:

$$E(t) - E_0 = J_0[\varphi(0) - \varphi(t)] = J_0[1 - \varphi(t)]. \quad (8)$$

From (5)-(8) we obtain the condition when the particle total energy $E(t_1) = 0$ at a moment $t_1 > 0$:

$$\varphi(t_1) = \frac{m v_0^2}{2J_0} = \frac{v_0^2}{v_{\mathrm{m}}^2} \leq 1 \,. \quad (9)$$

At a moment $t_2 \geq t_1$ this particle can already leave this well (Fig.1) with the final speed v_f which are determined on the basis of relations (4) and (5) taking into account that at the moment $t = t_1$ its total energy $E(t_1) = 0$:

$$0.5 m v_f^2 = E(t_2) - E(t_1) = J_0[\varphi(t_1) - \varphi(t_2)] \,. \quad (10)$$

Let's assume that at the initial moment of time t=0, the considered particle with the velocity v_0 (7) is within the potential well (Fig. 1) with the coordinate x $(L \geq x \geq -L)$. Then it is not difficult to find the moment t_2 when the particle leaves this well:

$$t_2 = \left(\frac{2 \cdot L \cdot k + L - x \cdot v_0 \cdot |v_0|^{-1}}{|v_0|}\right). \quad (11)$$

The value k in (11) is the number of particle collisions with boundaries of the well (Fig. 1) during the localization time $t_2 > t \geq 0$ and is determined by the following formula:

$$k = ceil\left(\frac{x \cdot v_0}{2L \cdot |v_0|} + \frac{|v_0| \cdot t_1}{2L} - 0.5\right), \tag{12}$$

where the function $ceil(z)$ is the minimum integer number $\geq z$, and the time t_1 is found from (9). Now, from relation (10), we find the following expression for the final particle velocity v_f, taking into account its sign, i.e. exit direction from the potential well (Fig. 1):

$$v_f = J_0^{0.5} \cdot [\varphi(t_1) - \varphi(t_2)]^{0.5} \cdot v_0 \cdot |v_0|^{-1}(-1)^k. \tag{13}$$

3. DISCUSSION OF RESULTS

Further, we will carry out research on the example of the following time dependence $\varphi(t)$ (2):

$$\varphi(t) = exp[-(t/T)^n] , \qquad (t \geq 0, n > 0) , \tag{14}$$

where T is some characteristic time interval.

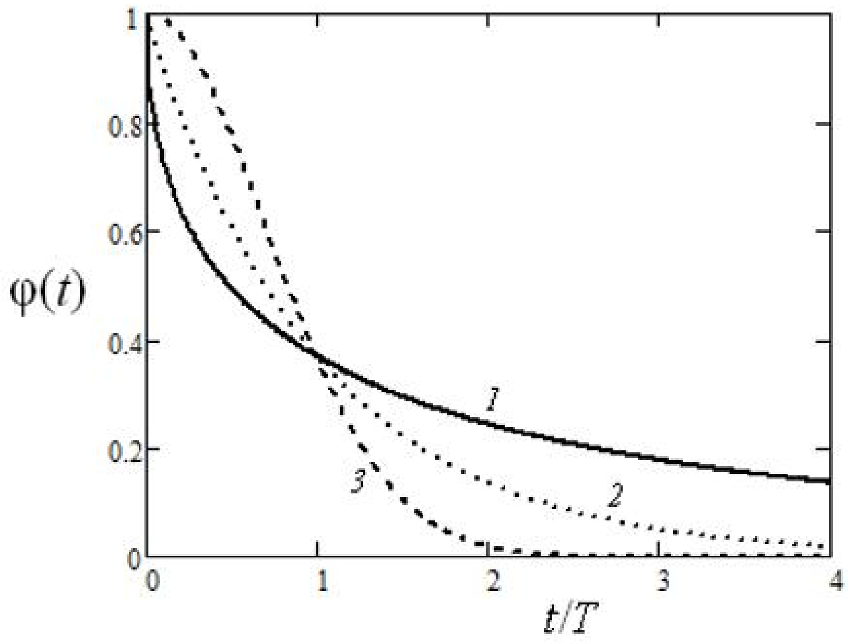

Fig.2. Dependence of the function φ(t) (14) on the time t (in units T) for the parameter n= 0.5 (curve 1), 1 (2) and 2 (3).

Fig. 2, for example, shows such dependences $\varphi(t)$ (14) for a number of the parameter n=0.5, 1 и 2. Based on the function(14), we obtain from (9) the moment t_1 at which the total energy of the particle $E(t_1) = 0$:

$$t_1 = T \cdot [ln(v_m^2/v_0^2)]^{1/n} . \tag{15}$$

Fig. 3 shows the dependences of the values v_f (13), t_2 (11), t_1 (15) and k (12) on the characteristic time interval T of the function $\varphi(t)$ (14). This parameter T in Fig. 3 is presented in fractions of the characteristic time $\tau = (2L/v_m)$ of the passage of the localized particle between the boundaries of the potential well (Fig. 1) with the maximum possible speed v_m (7). It can be seen that the final velocity v_f of the delocalized particle undergoes damped oscillations with increasing T (Fig. 3a). The times t_1 (15) and $t_2 \geq t_1$ (11) increase monotonically with increasing T (Fig. 3b). We note that equal values $t_2 = t_1$ correspond to the moments when, according to (13), the velocity v_f=0 (Figs.3a,b). The value k, which determines the number of collisions of the localized particle with boundaries of the well (Fig. 1) during the time interval $t_2 > t \geq 0$, also increases with increasing T (Fig. 3c). The dependences presented in Figs. 3 do not qualitatively change when the initial coordinate x of the particle localized within the well (Fig.1) changes at the initial time t=0.

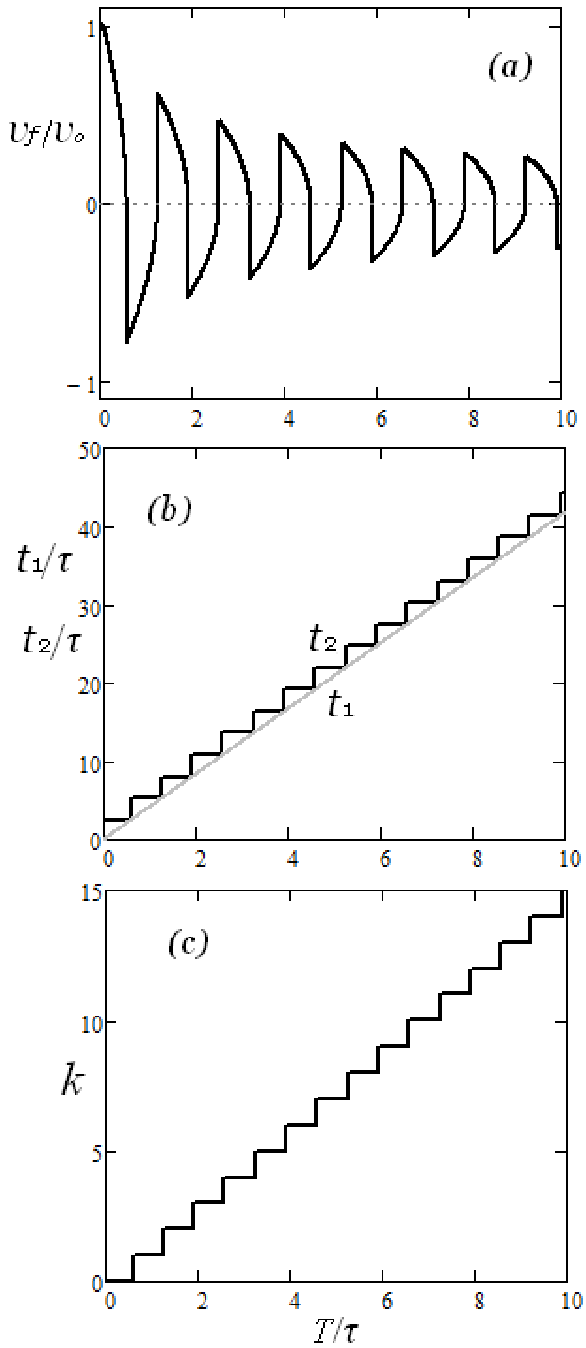

Fig.3. Dependences of values v_f (*a*), t_1, t_2 (*b*) and *k* (*c*) on the characteristic time interval *T* (in units $\tau = 2L/v_m$), when *n*=0.5, *x*=0.39*L*, and v_0= -0.36 v_m.

At the same time, the following characteristic σ is of special interest, which determines the ratio of the values of the kinetic energies of the particle averaged over the distance $L \geq x \geq -L$ when it leaves the potential well with the final velocity v_f and during its previous localization in this well with the speed $|v_0|$:

$$\sigma = \frac{1}{2L}\int_{-L}^{L}\left(\frac{v_f(x)}{v_0}\right)^2 dx, \qquad (16)$$

where the dependence $v_f(x)$ is determined on the basis of relations (9)-(13).

Fig. 4 shows the dependences of the value of σ (16) on the characteristic time interval *T* for three values of the parameter *n* = 0.5, 1 and 2 of the function $\varphi(t)$ (14). It can be seen that σ decreases monotonically with increasing *T*, starting from the value σ =1 at *T* =0, and can reach very small values σ <<1 at $T >> \tau = (2L/v_m)$. In this case, a faster decrease in σ takes place for the parameter *n* = 0.5, when there is a slower decrease with time *t* of the function $\varphi(t)$ (14) compared with the cases *n* = 1 and 2 (Fig. 2).

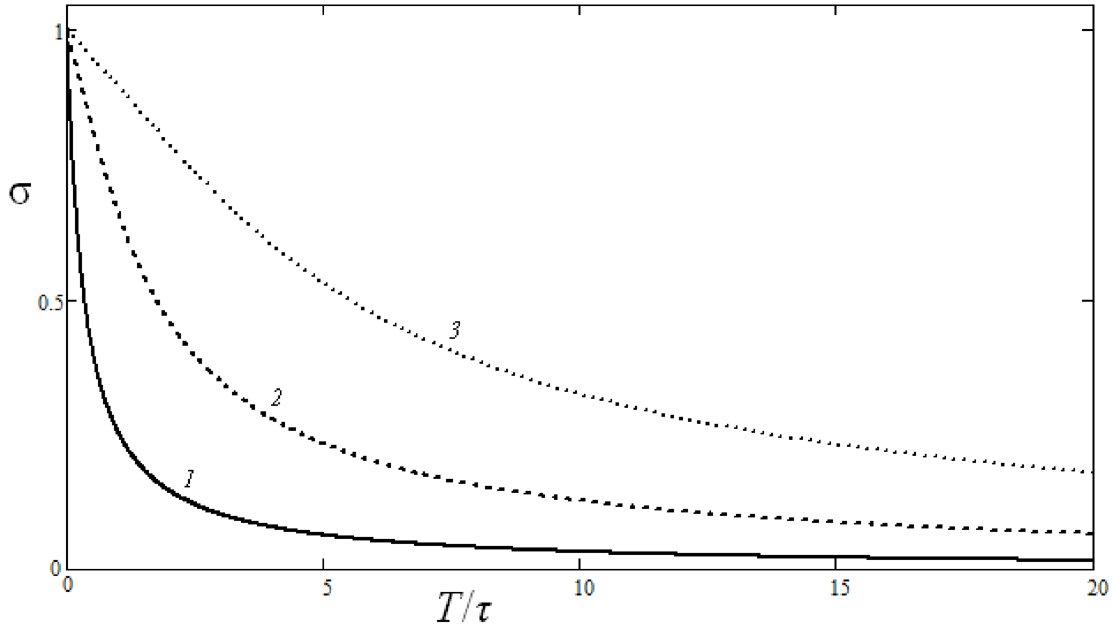

Fig.4. Dependence of the value σ on T (in units $\tau = 2L/v_m$) for the parameter *n*=0.5 (curve 1), 1 (2) and 2 (3), when v_0= -0.36 v_m.

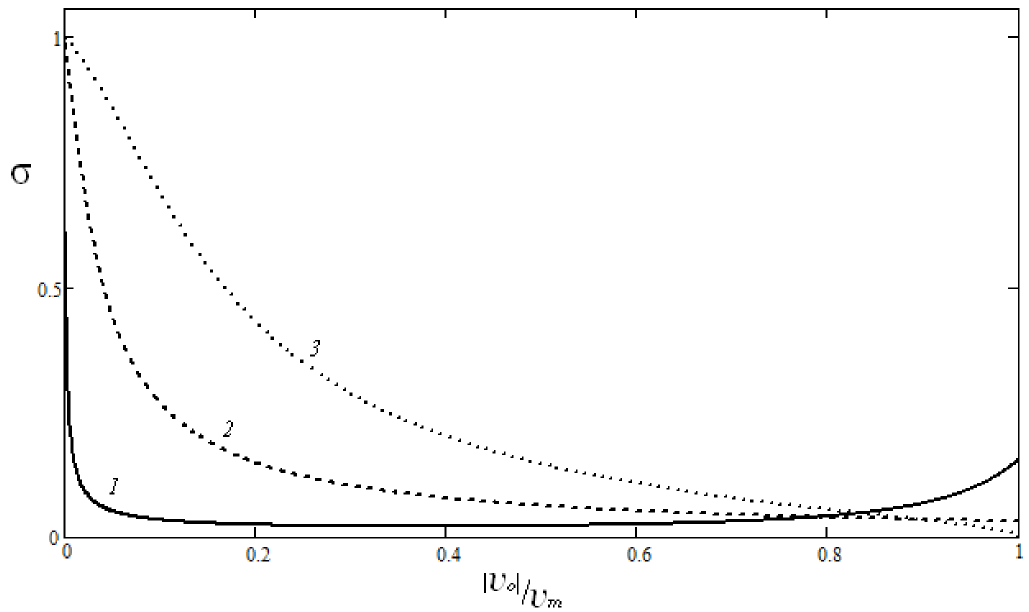

Fig.5. Dependence of the value σ on $|v_0|$ (in units v_m) for the parameter *n* =0.5 (curve 1), 1 (2) and 2 (3) when $T = 15\tau = 30L/v_m$.

Fig. 5 shows the dependences of the value σ (16) on the speed $|v_0|$ of the localized particle for the fixed time interval *T* and three parameters *n* = 0.5, 1, and 2 of the function $\varphi(t)$ (14). It can be seen that for *n* =1 and 2, the value of σ decreases with increasing $|v_0|$ (starting from the value σ =1 at v_0=0) over the entire interval $|v_0| \le v_{\mathrm{m}}$ and can reach very small values σ <<1 as $|v_0|$ approaches to v_{m} (curves 2 and 3 in Fig. 5). At the same time, at *n* = 0.5, the dependence $\sigma(v_0)$ after a sharp decrease with increasing $|v_0|$ may rise as $|v_0|$ approaches to v_{m} (curve 1 in Fig. 5).

Thus, based on the study, we can conclude that the efficiency of deceleration (cooling) of particles during their delocalization from the considered potential well (Fig. 1) increases with a slower decrease in the depth of the well, as well as with a reduction in its length $2L$, since the characteristic time interval T of the function $\varphi(t)$ (14) is presented on the horizontal axis in Figs. 3 and 4 in fractions of the value $\tau = (2L/v_m)$. According to Fig.3c, during this process, quite a lot of oscillations of the particle between the boundaries of the well should occur.

The established features and qualitative results of the delocalization process of particles from the considered rectangular potential well (Fig. 1) are also confirmed by additional calculations of the author carried out for various dependences of the well depth decrease with time t, different from the function $\varphi(t)$ (14).

[1] *C. Gonzalez-Ballestero, M. Aspelmeyer, L. Novotny, R. Quidant, and O. Romero-Isart.* Levitodynamics: Levitation and control of microscopic objects in vacuum. Science, **374** (6564),eabg 3027 (2021).

[2] *J. Millen, T.S. Monteiro, R. Pettit, and A.N. Vamivakas*. Optomechanics with levitated particles. Reports on Progress in Physics, **83**(2), 026401 (2020).

[3] *M. Weidemuller, C. Zimmermann*. Cold atoms and molecules: Concepts, Experiments and Applications to Fundamental Physics, Willey (2009).

[4] *A.Ch. Izmailov*. Trapping of classical particles by an electromagnetic potential well deepening over time. Optics and Spectroscopy, **119**, N 5, pp. 883-886 (2015).

[5] *A.Ch. Izmailov*. On the motion control of microparticles by means of an electromagnetic field increasing with time for spectroscopic applications. Optics and Spectroscopy, **122**, N2, pp. 315-321 (2017).

[6] *A.Ch. Izmailov*. On slowdown of particles in a vacuum at their delocalization from an electromagnetic well with a depth decreasing with time. arXiv:2209.10523 (presented 17 September 2022).

[7] *L.D. Landau and E.M. Lifshitz*. Course of Theoretical Physics, Vol. 1 : *Mechanics*, 3rd ed. (Butterworth-Heinemann, 1976).