I-V CURVE OF GRANULAR SUPERCONDUCTOR USING LONG JOSEPHSON JUNCTION APPROACH

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We calculate I-V curve of granulated superconductor using magnetic-field dependent $I_c(H)$ patterns for junctions with identical, thick, periodically arranged defects. We use corresponding analytical expression for a wide range of parameters, due to increased characteristic length in such structures. The field dependence of the critical current density and pinning potential are presented. The results are in agreement with another calculations.

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INTRODUCTION

Granular superconductivity was discovered many years ago in study [1] (see also [2]). With discovery of new classes of superconducting compounds [see, Ref. 3,4] interests to study of granular compounds becomes more actual. In study [5] authors investigate the electronic specific heat C as a function of temperature T. Model developed in this paper related with mesoscopically disordered s-wave superconductor and treated as a spatial ensemble of domains with continuously varying superconducting properties. Calculation of the critical current density jc of granulated superconductors (regarded as an assembly of superconducting grains being connected by weak (Josephson links) is complex problem. When solving this problem in a general case one should take into account not only the spread of the coupling energy, over Josephson junctions but also the correlation of the order parameter phases in different grains. The task may be significantly simplified if we neglect the latter: in this case currents in adjacent contacts may be regarded as mutually independent.

The model of the field dependence of the critical current density in superconducting compounds takes into consideration both possible mechanisms. Recent progress in technology makes it is possible to fabricate Josephson [6-7] with highly nonuniform critical current density, in which artificial defects, of virtually any desired strength and geometry, areadded to the junction barrier [8-9]. Most studies explore the properties of junctions with periodic arrangements of linear (columnar) defects which have the full width of the junction. It is clear that the understanding of the behaviour of magnetic-field Ic(H) patterns for Josephson junctions with nonuniform critical current density is important. It is use important to get an analytical expression for some model with critical current densities, which captures some of the primary qualitative features seen in experiments. Here, we

adapted a simple calculation an analytical magneticfield Ic(H) expression for a long Josephson junction with periodically alternating piecewise constant critical current density, within the uniform magnetic field approximation [10]. It is useful to note that anharmonic effect in current-phase relation [11,12] was considered in study [13].

BASIC EQUATIONS

Calculation of I-V curve of granular superconducting compounds at currents exceeding the critical current Icr is complicated by the fact that the resistive elements of such a system (intergrain junctions) are essentially nonlinear. It can be easily seen from a simple and frequently used model which regards an high-temperature superconductors and related compounds ceramic as a set of ID "threads" connected in parallel and composed of a large number of weak links connected in series. In this case the voltage V at the ends of a "thread" with a current I is shown in Fig. 1.

 $\begin{array}{cccc} The & voltage & drop & V & across & a \\ Superconductor/Normal & metal/Superconductor \\ junction with critical current & I_c & and normal resistance \\ R_N & is described by the function [14] \end{array}$

$$V(i) = N_k v_c \int_0^i \sqrt{(i/i_c)^2 - 1} f(i_c) di_c \qquad (1)$$

where i is the dimensionless current flowing through the junction. Distribution function of critical current f(ic) is determined by technological processes and

external magnetic field. Since for nonlinear systems the superposition principle is invalid the utilization of welldeveloped percolation and effective medium methods [14] is inapplicable for the calculation of the conductance of such a system.



Fig1 Granular superconducting model.

The normal resistance r_N of a contact is proportional to its thickness d_N and for "thick" contacts (see, for example [6]) depends only weakly i_c . If we neglect this dependence and use the distribution similarly to [14], we can get from Eq. (1) under linearization of I-V curve

$$V(i) = i^{n+2}, \ i < i0$$
 (2a)

$$V(i) = i, \ i \gg i0 \tag{2b}$$

As followed from last Eqs., a one-dimensional chain of Josephson links possesses a power-law I-V curve at moderate currents, is characteristic becoming linear for large currents. The natural current scale here is the maximum critical current of Josephson coupling. An attractive feature of the model in question is, apart from its simplicity, the feasibility of reconstructing the form of the junction distribution function $f(i_c)$ over the critical currents from the shape of the I-V curve (Ref. 14).

It is interesting to investigate the influence of external magnetic in I-V curve of granulated superconductor. It is well known, that the critical current of single short Josephson junction (the size of junction is small in respect penetration depth λj [6]) in extenal magnetic field changes as

$$\frac{I_{c}(H)}{I_{co}} = \left| \frac{\sin \frac{\pi \phi}{\phi_{0}}}{\frac{\pi \phi}{\phi_{0}}} \right|$$
(3)

Using Eq. (3) and (1) leads to result for I-V curve of granulated superconductors

$$V(i) = i^n, \ B = 0 \tag{4a}$$

$$V(i) = i^{0.5n}, B > Bc$$
 (4b)

However, this appoach seems oversimplified. In Ref. [10] author apply the analytical expression to Josephson junctions with thick identical periodically arranged defects, and compare it with numerical results obtained with the sine-Gordon equation. It was discussed the range of validity of the analytical Ic(H) pattern in terms of the increased characteristic length λ_{eff} . The dependence of λ_{eff} on the geometrical parameters is determined using qualitative arguments, and also self-consistently, from the length scale of soliton-type fluxon solutions of the sine-Gordon equation. It is clear that using more realistic form of function for critical current in the case of peridically modulated long Josephson junctions will lead to applicable results. Thus in this paper we develope the model granulated superconductor using magneticfield dependent $I_{c}(H)$ patterns for junctions with identical, thick, periodically arranged defects.

RESULTS AND DISCUSSIONS

In paper [10] a model dependence critical current density Jc(x) shown schematically in fig. 2, for a Josephson junction of length L, width W and penetration depth λj .



Fig. 2. Critical current density dependence in long junction with N=5

The critical current density alternates sequentially, taking two values J_1 and J_2 , which may differ both in magnitude and sign (n = 1,..., N). Thus, we have that $Jc = J_1$ within N intervals of length w, and $Jc = J_2$ within N intervals of length w, while the junction length is L = N. The calculated Ic(H) dependence in greneral case current distribution leads to result [10]

$$I_{c}(H) = LW \frac{\sin \pi \frac{\Phi}{\Phi_{0}}}{\pi \frac{\Phi}{\Phi_{0}}} \frac{1}{\sin \pi \frac{\Phi}{\Phi_{0}N}} \left\{ J_{1}^{2} \sin^{2} \pi \frac{r\Phi}{N\Phi_{0}} + J_{2}^{2} \sin^{2} \pi \frac{r'\Phi}{N\Phi_{0}} - J_{1}J_{2} \left[\sin^{2} \pi \frac{r\Phi}{N\Phi_{0}} - \sin^{2} \pi \frac{\Phi}{N\Phi_{0}} + \sin^{2} \pi \frac{r'\Phi}{N\Phi_{0}} + \right] \right\}^{1/2}$$
(5)

where the following notations were introduce

$$r = \frac{w}{l} r' = 1 - r = 1 - \frac{w}{l}$$
(6)

 $\frac{I_c(H)}{I_{co}} = \left| \frac{\sin \frac{\pi \Phi}{\Phi_0}}{\frac{\pi \Phi}{\Phi_0}} \right| \left| \frac{\sin \frac{\pi r \Phi}{N\Phi_0}}{r \frac{\pi \Phi}{N\Phi_0}} \right| \quad , \tag{7}$

For J₂=0 general Equation reduces to

with

$$I_{co} = A(rJ_1 + r'J_2)$$
(8)

The second factorin Eq. (7) in contrast to Eq. (3) is related to the collective interaction of the Josephson fluxons with arbitrary array of defects.

Changing of power exponent I-V curve in Eq. (4) taking into account modulated dependence critical current density in long Josephson junction versus of geometrical parameter presented in Fig. 3.



Fig.3. Power exponent n in I-V curve of granulated superconductor using long junction limit in respect to short junction limit n_0 versus geometrical factor.

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Decrasing of geometrical parameter r lead to the case short Josephaon junction limit. The zero-field critical current density in a defected junctions reduces with respect in a uniform junction with futher decreasing geometrical parameter r. As a result power exponent n in I-V curve of granulated superconductor using long junction limit also decreased in respect to short junction limit n_0 . The value of this pareameter in the case cuprate compounds is about 2.6. Magnetic field dependence of exponent n_0 was investigate in detail for cuprate compounds in [15]. Detail analysisin our case also shows that the power exponent n in the case long Josephson junction with modulated critical current density as 1/B, in similar way to short junction limit.

Thus, in this study it was investigated the I-V curve of granulated superconductor using magnetic-field dependent $I_c(H)$ patterns for junctions with identical, thick, periodically arranged defects. The analytical expression was used for a wide range of parameters, due to increased characteristic length in such structures. The field dependence of the critical current density is analyzed.

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