ON THE "FUZZY" ESSENCE OF THE MICROCOSM IN THE RADIATION OF AN ABSOLUTELY BLACK BODY: THE TWO-VALUED ARISTOTLE AND THE MANY-VALUED LUTFI ZADE OF LOGIC. PLANCK'S FORMULA

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This paper considers the radiation of a completely black body with a deeper look at the essence of the microworld, in which there is a connection between unusual quantum logic and fuzzy logic in Lutfi Zadeh's Fuzzy Set Theory . Max Planck used the concepts of combinatory to study black body radiation. What physicists call today the degree of degeneracy, statistical weight or thermodynamic probability for Boltzmann is his combinations without repetitions C_N^k,, for Gibbs his permutations with repetitions $L_N^{k,l,m}$, for Bose and Einstein combinations with repetitions A_P^N.. For Planck, too, combinations with repetitions of A, but with a completely different sound in them P and N and therefore A_N^P. In this, the fuzziness of Lutfi Zade's logic can be traced. There is no need to justify equal radiation quanta. They themselves come out of this new paradigm and the derivation of Planck's formula is simplified.

Key words: Combinatory, combinations with fnd with out repetitions, permutation, classical and quantum statistics, identically of atoms, statistics of Boltzmann, Gibbs, Bose – Einstein, Planck, black body radiation
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INTRODUCTION

It is well known that the Rayleigh-Jeans formula reflects the ultraviolet catastrophe (UV), which is unobservable in the experiment. From the point of view of classical physics, the Rayleigh-Jeans formula is correct. For a short wave, the number $k = \frac{2\pi}{\lambda}$ is a large value, which means a large number of vibrational degrees of freedom. If each degree has an equal energy *kT/2*, then short waves will have a very large radiation energy, which is the UK.Planck realized that the impeccable classical law of uniform distribution of energy over degrees of freedom does not hold here. Not all degrees of freedom of the short wave will receive energy for radiation.

So, at a given temperature T, inside the cavity of an absolutely black body there is a field of radiant energy of electromagnetic waves, and on its thin wall an oscillating atom emits waves of different lengths and frequencies, which Planck calls resonators. He says that each resonator carries a certain electrical charge. Interacting with the field of radiant energy, emitting or absorbing it, it changes the number of its oscillations. So the energy of the resonator U can increase or decrease over time. In classical physics, the thermodynamic interpretation itself requires an average value for it, because not all rapidly and randomly changing details of the processes of the microcosm are subject to study by a "macroscopic" observer. If there is a uniform distribution of energy over the degrees of freedom of the resonator, then this average U, regardless of the time of "macroscopic" observation t of the microworld, will be possible. And if there is no uniform distribution of energy, then, as Planck says [1]: "Even if the time of "macroscopic" observation t of the microworld can cover many periods of the processes of oscillation of the resonator, but still, if there is no uniform distribution of energy over degrees of freedom,

then this time interval t is quite small. Small so that the changes experienced during this interval by the observed quantities could be neglected". For example, the magnitude of black body radiation observed and measured by us. Indeed, we are not observing the resonator itself and its oscillation, but the beam that it emits. We observe rays from many resonators N. Due to the fact that the time interval of "macroscopic" observation of the microworld t is small, the energy of these rays cannot be averaged. It is in this, in the dissimilarity of the values of the energies of the rays, that Planck sees chaos, disorder, the entropy of the system S. He sees the set N of resonators with a total energy U_N and the set P of energy elements for them. Planck is asked: "In how many ways, that is, combinations with repetitions, can the distribution of P energy elements be distributed over N resonators, that is, the number of complexions, using the terminology of Boltzmann?" But Planck's complexions are fundamentally different from Boltzmann's complexions. For Boltzmann this question of Planck does not exist, for him there is only P=1 element of energy. This energy can have K of N resonators. For Boltzmann, statistics begins with the question, how many combinations without repetitions, that is, my complexions, will this result in? The more complexions, the more chaos and entropy in the system. The system tends to it at thermodynamic equilibrium. In this state, with a uniform distribution of energy over degrees of freedom, each complexion, each resonator in it, has a minimum of energy.

Today the Boltzmann complexions are called the thermodynamic probability W, and there is the Boltzmann principle for the measure of chaos, the entropy S:

$$S_N = k\log W \ , \qquad (1)$$

The Boltzmann complexion has become a tool for translating the knowledge of the combinatorics into the microworld.

THE CONCEPTION OF COMPLEXION

The word "complexion" in translation from English "complexion" means "complexion". That the state of the system with energy E, which the device distinguishes - this is the "face" of the system, that is, how many K molecules out of all N molecules are involved in the process. But how many such combinatorial combinations without repetitions K out of N are different "colors" of the same "face", which Boltzmann called the number of complexions.

Boltzmann begins his statistics with them. With the number of microstates, with degeneracy W, which lead to one macrostate of the system with energy E. The larger it is, the more chaos in the system and that it is more likely that it will occur at equilibrium. For Boltzmann, the number of his complexions is the number of combinations without repetitions:

$$W = C_N^k = \frac{N!}{k!(N-k)!} \qquad (2)$$

There is one energy value at which, for example, K molecules, "yes", have a given energy u, and (N - K) molecules, "no", do not have it. So many "yes +" and so many "no -" in terms of the classical two-valued logic of Aristotle. For example, out of 10 balls (atoms), the number of complexions will be maximum in the case of half and half "yes" and "no". 5 blue (fast) and 5 red (slow), because the number of combinations without repetitions, the number of complexions, its thermodynamic probability will be maximum in this case W=252.

In the thermodynamics this means its 2nd law. An isolated system will eventually come to thermodynamic equilibrium with a maximum of entropy, chaos. But to express this chaos, Boltzmann will say that the total energy of a system is divisible by the number of all its degenerate states, 2^N . This energy value is minimal. This is why the states of the system with lower energy are said to have a higher probability in the Boltzman distribution. Its maximum part will be taken by the most probable, that is, the most degenerate states. Therefore, the system chooses the minimum potential and maximum thermal energies.

So, for Boltzmann there is a value of the energy u and K of the molecules, "yes", they have it, and (N - K) molecules, "no", they do not. If he were also interested in other "qualities" of the atom, that is, what other energy u_2, u_3, u_4 , … received *l, m, ..* molecules from N molecules in the case of "no–", then here he would use a combinatorial formula for permutations of L with repetitions:

$$L_N^{k,l,m\cdots} = \frac{N!}{k!l!m!\dots} \qquad (3)$$

Boltzmann was not interested in this, but Gibbs was interested. That is why Gibbs believes that for him,

not combinations without repetitions C_N^k, as in Boltzmann, but precisely permutations with repetitions $L_N^{k,l,m..}$ is a thermodynamic probability or statistical weight. These are the Gibbs complexions. The two-valued logic of Aristotle remains in effect.

THE IDENTICALLY OF ATOMS

But Planck says for the first time: "How can permutations and arrangements of atoms matter, if they are identical to us in the invisible microworld?" Only combinations with repetitions matter for the statistics of the microworld. For example, out of 10 balls (atoms) it is possible that all balls are blue (with this energy), all balls are red (with a different energy), 1 blue 9 are red, etc. Variants, that is, the number of combinations with repetitions is $A_P^N = 11$. Note that combinations without repetitions for each variant are different, for example, out of 10 balls 2 are red $C_N^k = 45$. This is the meaning of the degeneracy of states in thermodynamics. Thus, of the three combinatorial measures - permutation, combination and placement - for Planck only combination with repetitions of A remains.

$$A_P^N = C_{N+P-1}^N = \frac{(N+P-1)!}{(P-1)!N!} \qquad (4)$$

This formula tells us that it is important to know how many states a system can have. The meaning of P is that even before combinations without repetitions, we announce the qualities of an atom, that is, energies E_1, E_2, E_3 …E_p. . For the balls (atoms) above, in their colors red and blue, (this energy and other energy) P=2.

So, the number P is the number of "qualities" of a micro-object to be in one state or another, and the number N is the "quantity" of the objects themselves. In combinatorics, the categories "quantity" and "quality" in terms of permutation, combination and placement are necessarily with the meaning that the same "quality" can simultaneously have the entire number of objects.

But not vice versa, that one and the same object has simultaneously all the "qualities" of the universe. As we can see, the two-valued logic of Aristotle is necessarily present in combinatorics. It is also present in Planck's vision of the identity of particles of the microworld. Hence, its combinations with repetitions A_P^N were useful to Bose-Einstein, and combinations without repetitions were useful to Fermi-Dirac in their derivation of the distribution of ideal quantum gas molecules. Today these are different statistics – classical and quantum. In them we distinguish between classical particles, fermions and bosons. We repeat that the logic of all these statistics remains classical, Aristotelian, two-valued.

ELEMENTS OF ZADEH'S FUZZY LOGIC

Then where in combinatorics to look for fuzzy logic and it is really present in quantum physics. Is it clear that Schrödinger's cat is both alive and dead at the same time? To this, physicists say that this is a superposition of states. We want to show that Planck

used fuzzy logic when deriving his formula for black body radiation. Therefore, he strongly opposed his quantum theory. He saw that in the statistics of identical particles of the microworld in combination with repetitions, "quantity" is replaced by "quality", and not A_P^N, as above for Bose-Einstein, but A_N^P.

All this does not look like Hegel's dialectic, in which there is a transition of quality into quantity and vice versa.

This is where fuzzy logic comes in

$$A_N^P = C_{N+P-1}^P = \frac{(N+P-1)!}{(N-1)!P!} \qquad (5)$$

As we can see, the difference in A_P^N and A_N^P is that P, being a category of the quality of a micro-object, for example, its energy, acquires the meaning of a category of quantity. This is exactly what Lorentz writes [1]: "Planck further calculates the number of ways in which it is possible to carry out the distribution of P elements of energy into N resonators." For example, 5 elements of energy at the same time in one resonator. Then A=1, all these elements of energy are in this resonator. Is it clear? Is this possible in classical physics? Of course not. Always P=1. An object can have only one quality and its contains either 1, or 2, .. or K, ... or N objects, that is, $A_N^1 = N$. They will have this quality, this given energy and not another.

This is the essence of the uniform distribution of energy over degrees of freedom. After all, we are talking about one quality of a micro-object. But how did Planck find himself in such a situation where P > 1 for him?

CHAOS FOR BOLTZMANN AND PLANCK

For identical atoms after combinations, the question of their placement and permutations does not make sense. This is exactly what Boltzmann and Planck believed. But for Boltzmann P=1. Boltzmann chooses one of $A_N^1 = N$, for example, K from N, and the meaning of degeneration, chaos appears for him there - these are combinations without repetitions C_N^k, that is, the number of complexions. So he translates combinatory into the world of atoms. For Boltzmann, an invisible microcosm with identical particles still allows him to combine identical objects taken at random without repetition. "I have chosen an object, it is already marked by me by this act of choice," Boltzmann would say. He can consider each separately any state of the system, because there is a uniform distribution of energy over degrees of freedom. He only needs to decide on chaos in the most chosen state, that is, with the number of complexions W. For Boltzmann, they are the degrees of freedom of the system. The system in the equilibrium state tends to the maximum number of complexins. Each complexion has a chance to "life", to receive its equal portion of energy for all and in balance, although it will be minimal. Indeed, this shows us the Boltzmann distribution. Lower energy states will always have a higher probability of being occupied. This is how Boltzmann approaches the study of his systems.

But not Planck. For Planck, initially chaos is in the very faces of the system, in its states. With a uniform distribution of energy over degrees of freedom, the system has only 1 person. But with its uneven distribution, not all complexions, but only some of them will receive their portion of energy, and this will be the concept of chaos for him. So, a high-frequency resonator, having a large number of degrees of freedom, will want more energy, but without receiving it, it will not be able to radiate. Therefore, UC is not observed in the experiment. For Planck, the essence of the microcosm is that with a limited supply of energy, those particles that are the most animated will eventually receive the least.

It is this that leads to a deviation from the uniform distribution of energy over the vibrational powers of the resonator. No more P=1. Planck sees chaos in this, the entropy of the system S. Therefore, the state of the system, that is, its face, which it acquires, is not unambiguous, chaos manifests itself in this and Planck notices it, introducing his own number of his complexions - the number of combinations with repetitions A.

This means a transition to a new level of understanding, that a micro-object can simultaneously have energies in 1ε, 2ε, 3ε,…, where ε is an energy quantum. This is the superposition of states that Planck came to and which is confirmed by Heisenberg's uncertainty principle.

RESULT

Planck did not like this very much, but he was forced to say: "It is necessary (in this position lies the core of quantum theory) to imagine U_N not as a continuous, indefinitely divisible quantity, but as a discrete quantity, consisting of an integer number of finite, equal parts. If we name one such part, the element of energy, as ε, then at the same time we get

$$U_N = NU = P\varepsilon \ ,$$

where R is a large integer. We leave the question of the value of ε open for the time being." He did not like it very much, and it was artificial for him that ε is an equal indivisible quantum of energy. Artificially, because it did not fit into the framework of the usual classical Aristotelian logic. But it fits into the framework of fuzzy logic in the theory of fuzzy sets of Lutfi Zade. In it, the measure of possibility, in contrast to the measure of probability, allows to be a superposition of states. As we can see, not as in the classical case A_P^N, but A_N^P . In A_P^N, our N objects are distributed to P qualities, but for Planck it is the other way around, P qualities are distributed on N objects, and it turns out the other way around A_N^P. Using the Stirling formula:

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n .$$

And limiting himself to an approximation sufficient for his purpose, Planck writes down the statistical formula as:

$$W = \xi = \frac{(N+P)^{N+P}}{N^N P^P}$$

As we can see, for counting in statistics it does not matter what to consider A_P^N or A_N^P. Therefore, for Bose and Einstein, without this depth, it is only important that the micro-objects are identical, and hence the combinations with repetitions A_P^N . Statistics are just statistics and their purpose is not to give an idea, but simply to count [3].

Nevertheless, the fact remains that here there is a distribution not of quantity over quality, but, on the contrary, a distribution of quality over quantity. Therefore, Planck called the theory quantum, here the number of quanta of energy. After all, he needed the quality P, which are elements of energy, each of which is ε, to imagine as the number of identical objects, that is, quanta. So, there are quanta ε, and their number P is 1ε, 2ε, 3ε, 4ε, … Pε . They can be represented as the number of states - energy levels. And there are N atoms or resonators - this is already a quality, and here, for example, not a number - 3 atoms, but a quality, that is, the 1st atom, the 2nd atom, the 3rd atom. The picture is not the same as in classical physics, that so many atoms have a given energy, but on the contrary, so many energy quanta have a given atom.

In the classical world, for example, the number of atoms with a given energy decreases, while in the quantum world the number of quanta decreases for a given atom. It goes to radiation in 1ε, 2ε, 3ε, 4ε, ... Pε. In principle, there can be no other portions of radiation. The fact that Planck really disliked the artificiality of an equal portion of ε is automatically removed from us. The derivation of Planck's formula is greatly simplified.

So, the Gibbs'canonical distribution, well known even before Planck, connects the number of atoms of one mode of vibration n with the energy E_n at a certain temperature T. For us, n takes on the meaning of counting the mode itself (1st, 2nd, etc.), the energy E_n takes on the meaning of energy quanta 1ε, 2ε, 3ε, 4ε, … Pε.

$$n = Ae^{\frac{-E_n}{kT}}$$
$$n + 1 = Ae^{\frac{-E_{n+1}}{kT}}$$

Where E_{n+1} is already a neighboring energy level. The difference between adjacent levels is the same in all cases n and it goes to radiation.

$$E_n - E_{n+1} = \varepsilon$$

$$\frac{n+1}{n} = 1 + \frac{1}{n} = e^{\frac{E_n - E_{n+1}}{kT}}$$

$$n = \frac{1}{e^{\frac{\varepsilon}{kT}} - 1}$$

If all these n atoms radiate according to the energy quantum ε, then the desired radiation energy U will take place.

$$U = \frac{\varepsilon}{e^{\frac{\varepsilon}{kT}} - 1}$$

This is the Planck's radiation law.

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