HIGGS BOSON DECAY CHANNELS $H\to\gamma\gamma$, $H\to\gamma Z$, $H\to gg$

S.Q. ABDULLAYEV, M.S. GOJAYEV, F.A. SADDIGH
Baku State University, AZ-1148, Z.Khalilov, 23, Baku, Azerbaijan
s_abdullayev@mail.ru, m_qocayev@mail.ru, f_seddig@yahoo.com

In the framework of Standard Model the process of scalar Higgs boson decay channels $H \to \gamma\gamma$ $H \to \gamma Z$, $H \to gg$ are investigated. It is shown that the processes $H \to \gamma\gamma$ and $H \to \gamma Z$ suppressed than the process $H \to gg$.

Keywords: Standard Model, Higgs boson, left and right coupling constants, spirality, Weinberg's parameter.
Pacs: 12.15-y; 12.15. Mm; 14.70. Hp.

The Weinberg-Salam unified theory of Electromagnetic and weak interaction (Standard Model – SM) has achieved great success [1]. It includes the production of neutral weak current, discovery of $W^{\pm}$- and Z^0-gauge bosons and some of its claims are investigated successfully in experiments. One of the important acclaims of SM is the prediction for the existence of scalar Higgs boson. Some experiments are carried out for the discovery of Higgs boson in different Experimental Labs.

Finally in LHC new information are received concerning the existence of Higgs boson with the mass of 125 GeV [2-5]. So the Higgs boson decay channels which give rise to Higgs bosons have got more attentions [6-9].

In this work we have investigated the channels of Higgs boson decaying to two gamma quanta, one gamma and one Z^0-boson, and to two gluon:

$$H \to \gamma + \gamma, \tag{1}$$

$$H \to \gamma + Z^0, \tag{2}$$

$$H \to g + g\,. \tag{3}$$

1. Higgs Decay to Two Photons.

Since photons are massless particles, they do not couple to the Higgs boson directly. Nevertheless, the $H\gamma\gamma$ vertices can be generated at the quantum level with loops involving massive particles which couple to the Higgs boson. The $H\gamma\gamma$ couplings are mediated by charged fermions and *W* boson loops.

The Feynman for the Higgs boson decay to photons with fermions loops are shown in the Fig. 1 the 4-momentum of particles are shown over the diagram).

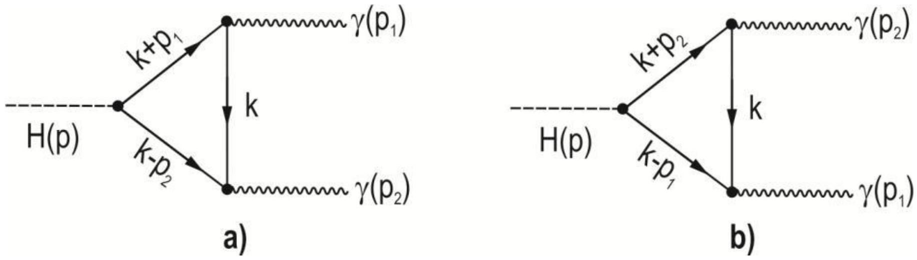

Fig. 1. The Feynman diagrams for the processes $H \to \gamma\gamma$

It is known that the interaction of Higgs boson with massive fermion is stronger, so we will calculate the diagram containing t-quark loop.

The matrix element for the first Feynman diagram could be written as follow:

$$M_a(H \Rightarrow \gamma\gamma) = -ie^2Q^2\frac{m}{\eta}a_\mu^{*(1)}a_\nu^{*(2)}\int\frac{d^4k}{(2\pi)^4}\frac{Tr[\gamma_\mu(\hat{k}+\hat{p}_1+m)(\hat{k}-\hat{p}_2+m)\gamma_\nu(\hat{k}+m)]}{(k^2-m^2)[(k+p_1)^2-m^2][(k-p_2)^2-m^2]}\,. \tag{4}$$

Here m and Q are the mass and charge of the fermion, $a_\mu^{*(1)}$ and $a_\nu^{*(2)}$ – are the 4-polarization vector of the fotons, $\eta = (\sqrt{2}G_F)^{-1/2} = 246\ GeV$ – is the vacuum expectation value of the Higgs boson field, G_F – signifies the Fermi constant of weak interaction.

For the sake of simplification of the matrix element, first we calculate the trace of Dirac matrices:

$$tr[\gamma_\mu(\hat{k}+\hat{p}_1+m)(\hat{k}-\hat{p}_2+m)\gamma_\nu(\hat{k}+m)] = 4m\{g_{\mu\nu}[m^2-k^2-(p_1.p_2)]+ \\ +4k_\mu k_\nu - 2k_\mu p_{2\nu} + 2k_\nu p_{1\mu} - p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu}\} \equiv 4mT_{\mu\nu}. \tag{5}$$

Now by using Feynman integral techniques we calculate the integral in equation (4)

$$\frac{1}{ABC}=\int_0^1 dx\int_0^1 dy\int_0^1 dz\delta(x+y+z-1)\frac{2}{[Ax+By+Cz]^3}. \quad (6)$$

We have $A=k^2-m^2$, $B=(k+p_1)^2-m^2$ and $C=(k-p^2)^2-m^2$ and we can simplify the denominator:

$$D=Ax+By+Cz=(k^2-m^2)x+[k^2+2(k\cdot p_1)-m^2]y+[k^2-2(k\cdot p_2)-m^2]z=$$
$$=(k^2-m^2)(x+y+z)+2(k\cdot p_1)y-2(k\cdot p_2)z=(k^2-m^2)+2(k\cdot p_1)y-2(k\cdot p_2)z=$$
$$=(k+p_1y-p_2z)^2+2(p_1p_2)yz-m^2. \quad (7)$$

We will consider here the case of $p_1^2=p_2^2=0$ and $x+y+z=1$. We define $b^2=m^2-2(p_1\cdot p_2)yz$, therefore, we can write D in the simplified form:

$$D=(k+p_1y-p_2z)^2-b^2. \quad (8)$$

Following this we can show that the integral in (4) becomes:

$$I_{\mu\nu}\equiv\int\frac{d^4k}{(2\pi)^4}\int_0^1 dy\int_0^{1-y}dz\frac{8mT_{\mu\nu}}{[(k+p_1y-p_2z)^2-b^2]^3}. \quad (9)$$

Let's change the variables under the integral by

$$k\Rightarrow k-p_1y+p_2z.$$

Then the integral (9) will be:

$$I_{\mu\nu}\equiv\int\frac{d^4k}{(2\pi)^4}\int_0^1 dy\int_0^{1-y}dz\frac{8mT'_{\mu\nu}}{(k^2-b^2)^3}. \quad (10)$$

At this case tensor $T'_{\mu\nu}$ will be equal to:

$$T'_{\mu\nu}\equiv g_{\mu\nu}(m^2-(p_1\cdot p_2))-g_{\mu\nu}(k-p_1y+p_2z)^2+4(k-p_1y+p_2z)_\mu\times$$
$$\times(k-p_1y+p_2z)_\nu-2(k-p_1y+p_2z)_\mu p_{2\nu}+2p_{1\mu}(k-p_1y+p_2z)_\nu-p_{1\mu}p_{2\nu}+p_{2\mu}p_{1\nu}. \quad (11)$$

It should be noted that the terms are – linear in k vanish when integrated (k is an odd function). Besides this the polarization vector for gamma quanta fulfill the following condition:

$$a_\mu^{*(1)}p_{1\mu}=0,\ \ a_\nu^{*(2)}p_{2\nu}=0.$$

As result the $T'_{\mu\nu}$ tensor will take a simpler forms:

$$T'_{\mu\nu}\equiv 4k_\mu k_\nu-k^2g_{\mu\nu}+p_{2\mu}p_{1\nu}(1-4yz)+g_{\mu\nu}[m^2-(p_1.p_2)(1-2yz)]. \quad (12)$$

Now by integrating over k we will get

$$\int\frac{d^4k}{(2\pi)^4}.\frac{4k_\mu k_\nu-k^2g_{\mu\nu}}{(k^2-b^2)^3}=\frac{i}{32\pi^2}g_{\mu\nu},$$
$$\int\frac{d^4k}{(2\pi)^4}.\frac{1}{(k^2-b^2)^3}=-\frac{i}{32\pi^2}\frac{1}{b^2}. \quad (13)$$

So we will have the following formula for the $I_{\mu\nu}$ tensor:

$$I_{\mu\nu}=\frac{im}{4\pi^2}[p_{2\mu}p_{1\nu}-(p_1.p_2)g_{\mu\nu}]I, \tag{14}$$

Here I represents the following integral

$$I=\int_0^1 dy\int_0^{1-y}dz\frac{1-4yz}{2(p_1\cdot p_2)yz-m^2}=\frac{1}{M_H^2}\int_0^1 dy\int_0^{1-y}dz\frac{1-4yz}{yz-m^2/M_H^2}=\frac{1}{M_H^2}\cdot I_0, \tag{15}$$

and we have considered that in the center of mass system $2(p_1.p_2)=M_H^2$.

Now we can write the matrix element for the Feynman diagram (a) as follow:

$$M_a(H\to\gamma\gamma)=\frac{e^2Q^2}{4\pi^2}\cdot\frac{m^2}{\eta M_H^2}\cdot a_\mu^{*(1)}a_\nu^{*(2)}[p_{2\mu}p_{1\nu}-(p_1.p_2)g_{\mu\nu}]I_0. \tag{16}$$

Following the same rule we can calculate also the Feynman diagram (b). Calculations show that the amplitude of the b) diagram is exactly the same as the first one $M_a=M_b$. The total amplitude is given by:

$$M(H\to\gamma\gamma)=\frac{e^2Q^2}{2\pi^2}.\frac{m^2}{\eta M_H^2}[(a^{*(1)}\cdot p_2)(a^{*(2)}\cdot p_1)-(p_1.p_2)(a^{*(1)}\cdot a^{*(2)})]I_0. \tag{17}$$

Let's now calculate the square of matrix element for $H\to\gamma\gamma$.

$$|M(H\to\gamma\gamma)|^2=\left(\frac{e^2Q^2}{2\pi^2}\right)^2\cdot\left(\frac{m^2}{\eta M_H^2}\right)^2\cdot|I_0|^2\,[(a^{*(1)}\cdot p_2)(a^{*(2)}\cdot p_1)-(p_1\cdot p_2)(a^{*(1)}\cdot a^{*(2)})]\times$$
$$\times[(a^{(1)}\cdot p_2)(a^{(2)}\cdot p_1)-(p_1\cdot p_2)(a^{(1)}\cdot a^{(2)})]. \tag{18}$$

The 4-polarization vector of gamma quanta with circular polarization $a=(0,\vec{a})$ could be written as follow:

$$\vec{a}=\frac{1}{2}(\vec{\beta}+il[\vec{n}\vec{\beta}]).$$

Here $\vec{n}$ is the unit vector along the photon momentum, $\vec{\beta}$ is the unit vector perpendicular to photons momentum: $\vec{\beta}\perp\vec{n}$ and $l=+1\,(-1)$ signifies the right (left) polarization of the photon.

By integration with respect to phase volume in the center of mass frame we obtain the following expression for the total probability:

$$\Gamma(H\to\gamma\gamma)=\left(\frac{\alpha}{2\pi}\right)^2\frac{Q^4}{64\pi M_H}\left(\frac{m^2}{\eta}\right)^2|I_0|^2\,(1+l_1\cdot l_2). \tag{19}$$

Here $l_1=\pm1$ and $l_2=\pm1$ – are the circular polarizations of photons. We can see from (19) that the gamma quanta should have right $(l_1=l_2=+1)$ or left $(l_1=l_2=-1)$ polarization.

The right polarization of one photon and left polarization of the other is being prohibited and this is due to the conservation of total momentum in the $H\to\gamma\gamma$ reaction.

After summing over the polarization states of photon, the probability for the $H\to\gamma\gamma$ reaction gets the following form:

$$\Gamma(H\Rightarrow\gamma\gamma)=\left(\frac{\alpha}{\pi}\right)^2\frac{Q^4}{64\pi M_H}\left(\frac{m^2}{\eta}\right)^2|I_0|^2. \tag{20}$$

2. Higgs Decay to photon and *Z* boson.

This process also fulfill two Feynman diagrams depicted in figure 2.

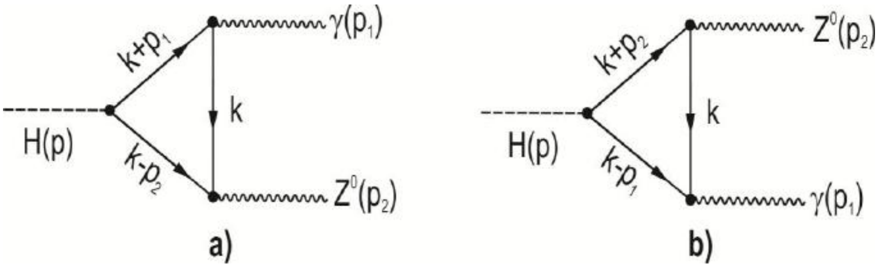

Fig. 2. Feynman diagrams for the process $H \Rightarrow \gamma Z^0$

Let's write first the matrix element for the Feynman diagram (a)

$$M_a(H \Rightarrow \gamma Z^0) = -ieQ\frac{m}{\eta}\frac{e}{2\sin\theta_w\cos\theta_w}a_\mu^* U_\nu \times$$

$$\times\int\frac{d^4k}{(2\pi)^4}\frac{Tr[\gamma_\mu(\hat{k}+\hat{p}_1+m)(\hat{k}-\hat{p}_2+m)\gamma_\nu[g_L(1+\gamma_5)+g_R(1-\gamma_5)](\hat{k}+m)]}{(k^2-m^2)[(k+p_1)^2-m^2][(k-p_2)^2-m^2]}. \quad (21)$$

Here a_μ^* and U_ν are the 4-polarization vectors of photon and *Z* boson and

$$g_L = \frac{1}{2}-\frac{2}{3}\sin^2\theta_w,\ \ g_R = -\frac{2}{3}\sin^2\theta_w \quad (22)$$

are the right and left coupling constants of t-quark to *Z* boson and θ_w is the Weinberg angle.

The trace of Dirac matrices will give:

$$Tr[\gamma_\mu(\hat{k}+\hat{p}_1+m)(\hat{k}-\hat{p}_2+m)\gamma_\nu[g_L(1+\gamma_5)+g_R(1-\gamma_5)](\hat{k}+m)] = 4m(g_L+g_R)\times$$
$$\times[g_{\mu\nu}(m^2-k^2-(p_1.p_2))+4k_\mu k_\nu-2k_\mu p_{2\nu}+2p_{1\mu}k_\nu-p_{1\mu}p_{2\nu}+p_{2\mu}p_{1\nu} \equiv 4m(g_L+g_R)T_{\mu\nu}. \quad (23)$$

While calculating the last relation the anti-symmetric term due to the indices μ and ν have been omitted because by summing over the polarization states of the Z^0 boson and photon the resulted tensor is symmetric. Like the case of $H \to \gamma\gamma$ we get the following expression for the matrix element:

$$M(H \to \gamma Z^0) = -2ieQ\frac{m}{\eta}\frac{e}{2\sin\theta_w\cos\theta_w}a_\mu^* U_\nu(g_L+g_R)I_{\mu\nu}. \quad (26)$$

Let's now find the square of the matrix element and sum over the polarization of particles:

$$\sum M(H \to \gamma Z^0) = -2ieQ\left(\frac{m^2}{\eta}\right)^2\frac{e^2}{x_w(1-x_w)}(g_L+g_R)^2(p_1.p_2)^2\,|\,I\,|^2, \quad (27)$$

Here $x_w = \sin^2\theta_w$ is the Weinberg parameter.

The total probability of the process $H \to \gamma Z$:

$$\Gamma(H \to \gamma Z^0) = \left(\frac{\alpha}{\pi}\right)^2 Q^4\left(\frac{m^2}{\eta}\right)^2 \cdot \frac{(g_L+g_R)^2}{32\pi M_H x_w(1-x_w)} \cdot |\,I_0\,|^2 \,. \quad (28)$$

It should be mentioned that diagrams for the processes $H \to \gamma\gamma$ and $H \to \gamma Z^0$ could be occurred by W-boson loops. The calculation for such diagrams will be considered in another work.

3. Higgs Decay to Two Gluons.

One of the other interesting decay channel for Higgs boson is the process of converting to two gluons. The diagrams for this are shown in fig. 4.

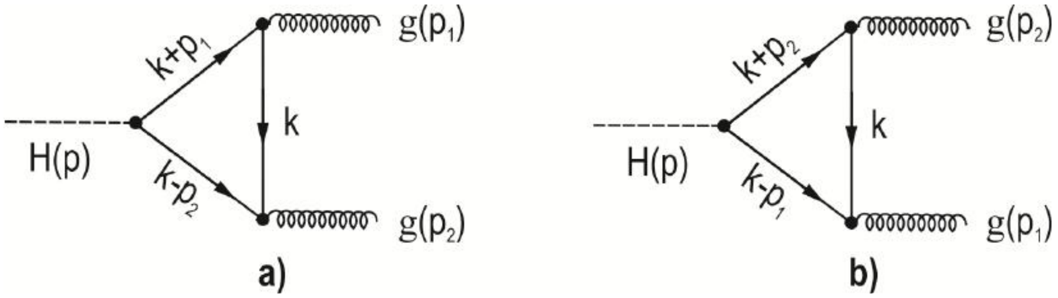

Fig. 3. Feynman diagrams for the process $H \to gg$.

Let's first write the matrix element for the diagram a:

$$M_a(H \to gg) = -ig_s^2 \frac{m}{\eta} \varepsilon_\mu^a \varepsilon_\nu^b Tr\left(\frac{\lambda_a}{2}.\frac{\lambda_b}{2}\right) \int \frac{d^4k}{(2\pi)^4} \frac{tr[\gamma_\mu(\hat{k}+\hat{p}_1+m)(\hat{k}-\hat{p}_2+m)\gamma_\nu(\hat{k}+m)]}{(k^2-m^2)[(k+p_1)^2-m^2][(k-p_2)^2-m^2]}, \quad (29)$$

Here g_s is the constant for the quark-gluon interaction, ε_μ^a and ε_ν^b are the polarization vectors for gluons and λ_a are the Gell-Mann matrices.

We first find the trace of Gell-Mann matrices:

$$Tr\left(\frac{\lambda_a}{2}.\frac{\lambda_b}{2}\right) = \frac{1}{4} tr(\lambda_a \lambda_b) = \frac{1}{2}\delta_{ab}.$$

Now we carry on the summation over the color and polarization of the gluons:

$$\sum_{a,b} \delta_{ab}\delta_{ab} = \sum_a \delta_{\alpha a} = 8, \quad \sum_{pol} \varepsilon_\mu^a \varepsilon_\rho^{*a} \varepsilon_\nu^b \varepsilon_\sigma^{*b} = g_{\mu\rho} g_{\sigma\nu} . \quad (30)$$

Other calculations are carried like the case of the $H \to \gamma\gamma$. As result the square of the transition amplitude for the case of $H \to gg$ is:

$$\sum_{pol} |(H \to gg)|^2 = 4\left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{m^2}{\eta}\right)^2 |I_0|^2 . \quad (31)$$

Here $\alpha_s = \frac{g_s^2}{4\pi}$ is the constant for the strong interaction.

So, we find here the total probability of the decay of scalar Higgs boson to two gluons:

$$\Gamma(H \to gg) = \frac{1}{8\pi}\left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{m^2}{\eta}\right)^2 \frac{|I_0|^2}{M_H}. \quad (32)$$

Let's now compare the probability of the decay of Higgs boson to two gluon over the decay to two photons:

$$\frac{\Gamma(H \to gg)}{\Gamma(H \to \gamma\gamma)} = \left(\frac{\alpha_s}{\alpha}\right)^2 . \frac{8}{Q^4}. \quad (33)$$

The electric charge for the t-quark is 2/3, according to last calculations when we take the value of $\alpha_s(M_z) = 0.114 \pm 0.0007$ for the strong interaction and $1/\alpha(M_z) = 127.916 \pm 0.015$ for the case of electromagnetic interaction the following value will be derived for the ratio of probability which is quite high:

$$\frac{\Gamma(H \to gg)}{\Gamma(H \to \gamma\gamma)} \approx 8600.$$

The real reason for this is the fact that the coupling constant for the strong interaction is much bigger (stronger) than electromagnet interaction $(\alpha_s >> \alpha)$.

[1] *S.Q. Abdullayev.* Lepton-lepton və lepton-hadron qarşılıqlı təsirlərdə zəif cərəyan effektləri. I Hissə. Bakı, «AM965 MMC» nəşr., 2012, 482 səh.
[2] ATLAS Collaboration. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. Phys. Lett., 2012, B716, p.1-29.
[3] ATLAS Collaboration. Combined search for the Standard Model Higgs boson in pp collision at $\sqrt{s} = 7$ *TeV* with the ATLAS detector. Phys. Rev., 2012, D86, 032003-1-31.
[4] The CMS Collaboration. Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. 2012, arXiv 1207.7235.
[5] The CMS collaboration Combined of searches for the Standard Model Higgs boson in pp collision at $\sqrt{s} = 7$ *TeV*, 2012, arXiv: 1202.1488, v1.
[6] *W.J. Marciano, C. Zhang, S. Willenbrock.* Higgs Decay to two photons. arXiv: 1109.5304 V2, 2011.
[7] *M. Shifman, A. Vainshtein, M.B. Voloshin, V. Zakharov.* Higgs decay into two photons through the W-boson loop; arXiv: 1109.1785, v3, 2011.
[8] *Da Huang, Tang Y. Wu.* Note on Higgs Decay into two photons $H \to \gamma\gamma$: 2012, arXiv: 1109.4846, v2.
[9] *S.Q. Abdullayev, F.A. Saddigh.* Bakı Universitetinin Xəbərləri, fizika-riyaziyyat elmləri seriyası, 2014, №1, s.142-151.

Received: 24.12.2014