TRAPPING OF CLASSICAL PARTICLES BY A POTENTIAL WELL DEEPENING WITH TIME

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The new trapping mechanism of sufficiently slow-speed particles by a potential well deepening with time is established from basic relations of classical mechanics. We consider situations when such particles are in high vacuum and external forces acting on given particles are not dissipative. Corresponding potential wells may be induced by a controllable electromagnetic field with nondecreasing strength and fixed spatial distribution. Detailed analysis of features of corresponding traps of particles is carried out on the visual example of the one-dimensional rectangular potential well. Obtained results may be used for motion control and high resolution spectroscopy of charged and neutral particles.

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1. INTRODUCTION

Electromagnetic traps for free charged and neutral particles without material walls allow to localize and observe these particles during a comparatively long period of time thereby creating conditions for detailed research of their properties [1]. In particular, such traps of microparticles in the high vacuum open new possibilities for contactless measurements of forces acting on given particles with extremely high accuracy and allow their micromanipulations [2].

In the present work we establish and analyze the sufficiently universal mechanism of trapping and localization of classical particles in a potential well, which is induced by an electromagnetic field with a nondecreasing strength and fixed spatial distribution. It is important to note that we consider situations when given particles move in a high vacuum without friction. Depending on whether particles have electric (magnetic) moment, it is possible to use the controllable electric (magnetic) field or nonresonance laser radiation for their analyzed trapping. We establish general conditions for such trapping of sufficiently slow-speed particles from basic relations of classical mechanics (section 2). Detailed analysis of features of corresponding electromagnetic traps of particles is carried out on the visual example of the one-dimensional rectangular well deepening with time (section 3). In conclusions we discuss possible generalizations and applications of obtained results for motion control and high-resolution spectroscopy of various microparticles (section 4).

2. BASIC RELATIONS

Let us consider a point particle with the mass m freely moving in a three-dimensional space before its entering to the region V of the potential well $U(\boldsymbol{r},t)$, which explicitly depends not only on the coordinate $\boldsymbol{r}$ but also on time t. The total energy of such a particle with the non-relativistic velocity $\boldsymbol{v}$ is described by the known formula [3]:

$$E(\boldsymbol{r},\boldsymbol{v},t) = 0.5mv^2 + U(\boldsymbol{r},t). \qquad (1)$$

Further we will consider the potential energy $U(\boldsymbol{r},t)$ of the following form:

$$U(\boldsymbol{r},t) = \sigma(\boldsymbol{r}) * \varphi(t), \qquad (2)$$

where the coordinate function $\sigma(\boldsymbol{r}) \leq 0$ in the region V, and $\varphi(t) \geq 0$ is the nondecreasing function of time t. Such a potential well (2) for particles may be created by a controllable electromagnetic field with the growing strength (up to a certain time moment) but with a fixed spatial distribution [4]. In this case the motion equation [3] for the particle has the form:

$$m\frac{d^2\boldsymbol{r}}{dt^2} = -\varphi(t)\frac{d\sigma(\boldsymbol{r})}{d\boldsymbol{r}}. \qquad (3)$$

On the basis of relationships (1)-(3) we directly receive the following formula for the time derivative of the total energy $E(\boldsymbol{r},\boldsymbol{v},t)$:

$$\frac{dE}{dt} = \sigma(\boldsymbol{r})\frac{d\varphi(t)}{dt} \leq 0. \qquad (4)$$

Thus, according to (4), an increase of the function $\varphi(t)$ with time t leads to the decrease of the total energy $E(\boldsymbol{r},\boldsymbol{v},t)$ (1) of the particle in the region V of the potential well, where $\sigma(\boldsymbol{r}) < 0$. We see also from the formula (1) that the particle can not go beyond the potential well (where $U(\boldsymbol{r},t) = 0$), when its energy E will be negative. It is important to note that such a classical particle will be localized in the region V of the potential well even after output of the nondecreasing function $\varphi(t)$ on a constant maximum value, when a negative total energy $E < 0$ of this particle will be conserved. It is obvious, that sufficiently fast particles will not be captured in the considered trap. Detailed analysis of dynamics of particles may be carried out for electromagnetic traps with definite spatial configurations. In the next section we will establish a number of important features of trapping and localization of sufficiently slow-speed non-relativistic particles on the visual example of the one-dimensional rectangular potential well.

3. ONE-DIMENSIONAL RECTANGULAR POTENTIAL WELL

Let us consider a point particle with the mass m freely moving with the velocity $v_0 > 0$ along the axis x (Fig.1) from the region $x < -L$ and in a certain moment t reaching the boundary $x = -L$ of the following potential well:

$$U(x,t) = -J_0 * \varphi(t) * \eta(L^2 - x^2), \quad (5)$$

where $J_0 > 0$ is the constant value with the energy dimension, $1 \geq \varphi(t) \geq 0$ is the nondecreasing function of time t, $\eta(y)$ is the step function ($\eta(y) = 1$ for $y \geq 0$ and $\eta(y) = 0$ if $y < 0$). From (3) we receive the motion equation of the particle for the potential well (5):

$$m\frac{d^2x}{dt^2} = J_0 * \varphi(t) * [\delta(x+L) - \delta(x-L)], \quad (6)$$

where $\delta(y)$ is the Dirac delta-function. According to Eq.(6), an abrupt increase of the particle velocity occurs from the initial value v_0 to $v \geq v_0$,when this particle falls into the well (5) in the moment t. Connection between given values v_0 and v is determined from the formula (1) for the total energy of the classical particle in this moment t:

$$E(-L,t) = 0.5mv_0^2 = 0.5mv^2 - J_0 * \varphi(t), \quad (7)$$

According to Eq.(6), the considered particle further will move inside the rectangular well (5) with the constant velocity v and will reach the opposite well boundary (with the coordinate $x = L$) in the moment $(t + 2L/v)$. However this particle will not be able to overcome the potential well if its total energy E (1) will become negative because of an increase of the function $\varphi(t)$ (7) with time t, that is at the following condition:

$$0.5mv^2 - J_0 * \varphi(t + 2L/v) < 0. \quad (8)$$

Under condition (8), the particle is reflected from the well boundary with the coordinate $x = L$ and will move in the reverse direction with the constant velocity $(-v)$ up to arrival to the opposite well boundary (with the coordinate $x = -L$) in the moment $(t + 4L/v)$. Because of the relationship $\varphi(t + 4L/v) \geq \varphi(t + 2L/v)$, the similar motion of the particle with the velocity v from the boundary $x = -L$ of the potential well up to $x = L$ will be repeated and so on.

Thus, according to condition (8), the maximum possible speed $v_{max}(t)$ of particles, captured in the considered trap (5) in the moment t , is determined by the equation:

$$0.5mv_{max}^2 = J_0 * \varphi(t + 2L/v_{max}). \quad (9)$$

After finding of the value $v_{max}(t)$ from Eq.(9), we obtain from formula (7) the maximum speed $\tilde{v}_0(t)$ of a free particle, which, after falling into the potential well (5) in the moment t, will be localized in this well:

$$\tilde{v}_0(t) = \sqrt{v_{max}^2(t) - \frac{2J_0\,\varphi(t)}{m}}. \quad (10)$$

The minimum speed $v_{min}(t)$ of particles, captured in the considered trap in the moment t, is determined from formula (7) at the value $v_0 = 0$:

$$v_{min}(t) = \sqrt{\frac{2J_0\,\varphi(t)}{m}} \quad (11).$$

Let us assume, that the function $\varphi(t)$ in (5) increases up to the maximum value 1 during the period from 0 to T and then will be constant, that is $\varphi(t) = 1$ if $t \geq T$. In this connection we introduce following characteristic values for the potential well (5) with dimensions of speed and energy:

$$w = 2L/T, \qquad K = 0.5mw^2. \quad (12)$$

In case of the comparatively shallow well (5), when $J_0 \leq K$, we receive from Eq.(9) the maximum speed $v_{max}(t)$ of particles captured in the trap, which is constant during the period $0 \leq t \leq T$ of growth of the function $\varphi(t)$:

$$v_{max}(t) = v^* = \sqrt{\frac{2J_0}{m}} = w * \sqrt{\frac{J_0}{K}}. \quad (13)$$

Then we obtain the corresponding allowed initial speed of a free particle from (10):

$$\tilde{v}_0(t) = \sqrt{\frac{2J_0}{m}[1 - \varphi(t)]}\ , \ (0 \leq t \leq T). \quad (14)$$

In case of a sufficiently large depth of the potential well (5), when $J_0 > K$, according to Eq.(9), the speed $v_{max}(t)$ of particles, captured in the trap, reaches the constant maximum value v^* (13) in the time period $t^* \leq t \leq T$, where

$$t^* = T - 2L * \sqrt{\frac{m}{2J_0}} = \left(1 - \sqrt{\frac{K}{J_0}}\right)T. \quad (15)$$

Further, for definiteness, we consider the following time dependence $\varphi(t)$ of the potential (5):

$$\varphi(t) = \left(\frac{t}{T}\right)^n \eta(T - t) + \eta(t - T), \quad (n > 0,\ t \geq 0). \quad (16)$$

According to formula (16), the depth of the potential well (5) increases from 0 to J_0 during the period $0 \leq t \leq T$, and will have the maximum constant value J_0 when $t > T$.

Fig.2 presents dependences of speeds $\tilde{v}_0(t)$, $v_{max}(t)$ and $v_{min}(t)$ (9)-(11) of trapped particles on time t for 2 parameters n=0.5 and 2 of the given function $\varphi(t)$ (16). We see that the considered potential well (5) carries out trapping and localization of free particles, whose speeds $|v_0|$ in a moment t of its falling in the well is restricted by the value $\tilde{v}_0(t) > |v_0|$ from relationships (9), (10). Then the speed $|v|$ of given captured particles inside the well is between values $v_{max}(t)$ and $v_{min}(t)$ (Fig.2) determined by (9), (11). Trapping of new particles in the potential well (5), (16) stops after the moment T, when this well will be stationary (Fig.2). However classical particles, captured in the given electromagnetic trap before the moment T, will remain there also in following time $t > T$. Possible speeds $|v|$ of these particles, remained inside the potential well, will be between 0 and $\sqrt{2J_0/m}$. According to Fig.2, speed intervals of particles, captured in such an electromagnetic trap in fixed moments $t < T$, essentially depend on the parameter n of the function $\varphi(t)$ (16), that is on increase rate of the well depth with time t. At the sufficiently shallow potential well (5), when $J_0 \leq K$, corresponding speeds of particles in Fig.2a,b are described by simple formulas $v_{min}(t)$ (11), $v_{max}(t) = v^*$ (13) and $\tilde{v}_0(t)$ (14). In case $J_0 > K$, increase of the speed $v_{max}(t)$ occurs in the time period $0 \leq t \leq t^*$ (15) and the constant value $v_{max}(t) = v^*$ (13) establishes after the moment t^* (Fig.2c,d). In particular, for the given function $\varphi(t)$ (16), speeds $\tilde{v}_0(t)$ and $v_{max}(t)$ coincide in the initial moment $t = 0$ and, according to (9), (10), have the following form when $J_0 \geq K$:

$$v_{max}(t=0) = \tilde{v}_0(t=0) = \left(\frac{2J_0}{m}\right)^{\frac{1}{(n+2)}} \left(\frac{2L}{T}\right)^{\frac{n}{(n+2)}} = \left(\frac{J_0}{K}\right)^{\frac{1}{(n+2)}} w \,. \tag{17}$$

According to (17), available speeds of trapped particles increase with growth of the length $2L$ and the depth J_0 of the potential well (5), (16) and also with decrease of the characteristic time T of its deepening.

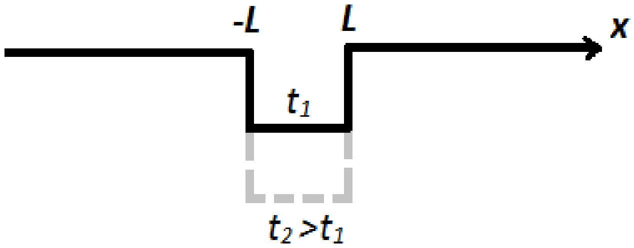

Fig.1. Scheme of the one-dimensional rectangular potential well deepening with time *t*.

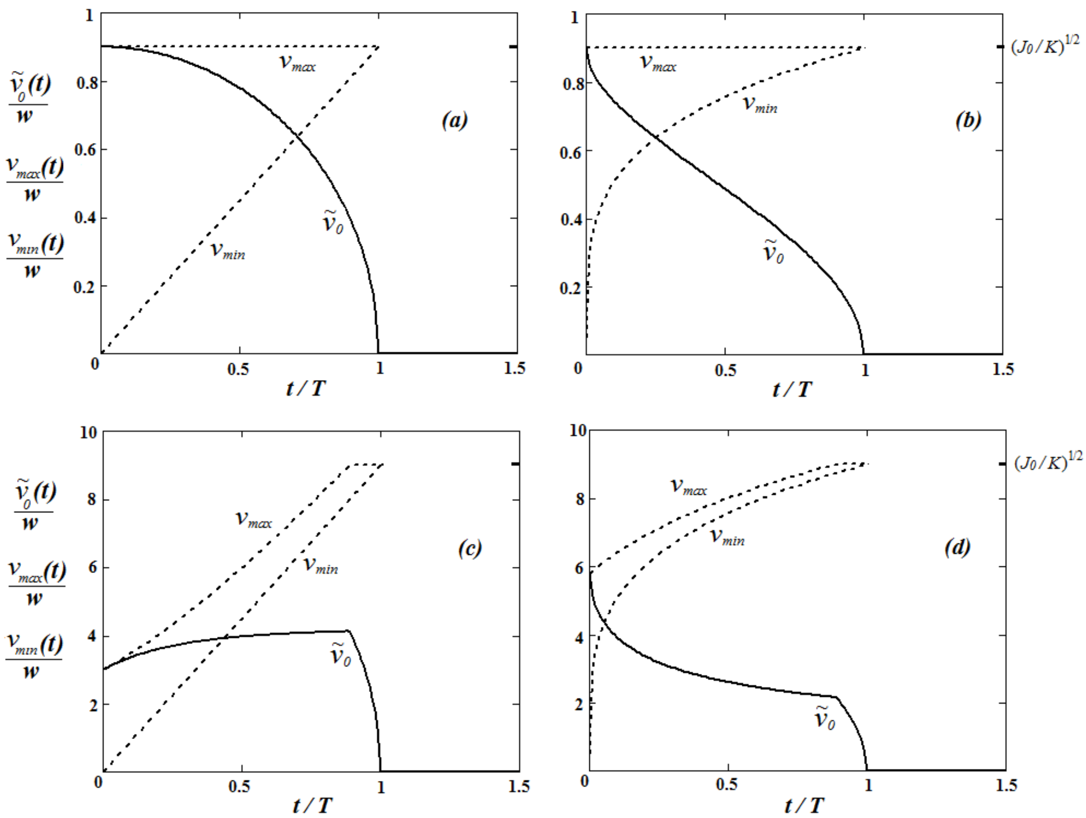

Fig.2. Speeds $\tilde{v}_0(t)$, $v_{max}(t)$ and $v_{min}(t)$ of trapped particles versus time t of their getting into the one-dimensional rectangular potential well (5) with the value $J_0 = 0.81K$ (a, b) and $81K$ (c,d) for the parameter n =2 (a, c) and 0.5 (b, d) of the function $\varphi(t)$ (16).

4. CONCLUSIONS

From basic relations of classical mechanics, we have shown possibility of trapping and localization of sufficiently slow-speed particles by a potential well deepening with time, which is described by the general formula (2). Such universal traps may be created in practice for various charged and neutral classical particles (having electric or magnetic moments) by means of the controllable electric or magnetic field with strength increasing during a certain period but at a fixed spatial field distribution. Similar potential wells may be induced also by amplifying nonresonance laser beams of definite spatial configurations.

In section 3 we have established a number of interesting features of given electromagnetic traps on the visual example of the one-dimensional rectangular potential well. Such a well may be created in practice by the controllable local homogeneous electric (or magnetic) field on the propagation path of a collimated beam of classical particles having electric (or magnetic) moment. During the definite time of growth of the given field strength (up to a certain maximum value), such an electromagnetic well will capture sufficiently slow-speed particles from the beam. Characteristic speeds of these trapped particles may be estimated on the basis of relations obtained in the present work.

Author have carried out numerical calculations also for some other potential wells of the type (2), including cases of two- and three-dimensional wells with cylindrical and spherical symmetries. These calculations confirmed following qualitative results (a), (b) and (c) of given work.

(a) Even a highly shallow but deepening with time potential wells of the type (2) will continuously capture sufficiently slow-speed particles.

(b) Such trapped classical particles will remain inside the potential well even after going out of the corresponding nondecreasing electromagnetic field on a stationary value. However this stationary well already will not capture new particles.

(c) Trapped particles will carry out nondamped oscillatory motions within a corresponding potential well because such wells are based on nondissipative forces in the high vacuum.

Of course, results obtained in the present work are valid only in the absence of an interaction between particles travelling through a potential well. However such an interaction may be essential at a sufficiently high concentration of captured particles in a comparatively small volume of an electromagnetic trap.

Considered electromagnetic traps will extend possibilities for motion control and high resolution spectroscopy of such noninteracting microparticles which move without friction in the high vacuum under action of the controllable electric (magnetic) field or nonresonance laser radiation.

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