DE SITTER COSMOLOGICAL MODEL AND THE PROBLEM OF DARK MATTER AND ENERGY

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It is shown that after contraction of unitary irreducible representations of the de Sitter group SO (4,1), a direct sum of unitary irreducible representations of the Poincaré group with different signs of the rest mass is obtained. This result is used to interpret the phenomena of "dark matter" and "dark energy" in terms of Wigner's elementary quantum systems in the de Sitter world.

Keywords: de Sitter world, $SO(4,1)$ group, Wigner-Inönü limit, "dark matter", "dark energy", contraction, elementary systems.

1. INTRODUCTION

Recently, the nature of dark matter and dark energy has been intensively studied, and various assumptions about their composition are being considered, [1].

In this paper, we propose an explanation of these phenomena by the Wigner's elementary systems in the de Sitter world, [2].

2. WIGNER'S THEORY OF ELEMENTARY SYSTEMS

A real understanding of the nature of the spin and mass of elementary particles appeared after the classical work of E. Wigner on the representations of the inhomogeneous Lorentz group (that is, the Poincarė group), [3]. He studied the projective representation of this group in the Hilbert space of states of a quantum system and introduced the concept of an elementary system and showed that such systems correspond to irreducible representations. Moreover, Wigner proved that the rest mass and the spin of an elementary particle are invariants that uniquely characterize irreducible representations of the Poincarė group.

Invariants of the inhomogeneous Poincarė group are constructed using translation generators P_μ and 4-dimensional rotations $M_{\mu\nu}$ as follows, [4]:

$$m^2 = P_\mu P^\mu, \qquad w^2 = w_\mu w^\mu = m^2 s(s+1)$$
$$w_\varrho = \frac{1}{2}\epsilon_{\lambda\mu\nu\varrho}P^\lambda M^{\mu\nu} \tag{1}$$

where m - mass, s - spin of the particle, and w_ϱ is the Pauli-Lubansky-Bargmann vector.

Important! In the case $m^2 > 0$, there is a third invariant - the sign of energy:

$$\varepsilon = \frac{P_0}{|P_0|} = \pm 1.$$

3. EINSTEIN'S EQUATIONS AND DE SITTER'S SOLUTIONS

According to general relativity theory, the Einstein equations in the vacuum, take the form, [5]:

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \tag{2}$$

where $R_{\mu\nu}$ - Ricci tensor, $g_{\mu\nu}$ - metric tensor, Λ - cosmological constant (the values of Λ are permanently refined with the accumulation of observations: $\Lambda \sim 10^{-53} m^2$, 1998).

It is known that general solutions of Einstein's equations do not have a group of motions. But in 1917 Willem de Sitter found two solutions of (2) for $\Lambda \neq 0$ that allow 10-parameter groups of motions, which are the maximal groups of motions of Einstein spaces, [5]:

$$ds^2 = \frac{dr^2}{1-r^2/R^2} + r^2(d\vartheta^2 + sin^2\vartheta d\varphi^2) - (1 - r^2/R^2)c^2dt^2, \quad if\ \Lambda > 0, \tag{3}$$

$$ds^2 = \frac{dr^2}{1+r^2/R^2} + r^2(d\vartheta^2 + sin^2\vartheta d\varphi^2) - (1 + r^2/R^2)c^2dt^2, \quad if\ \Lambda < 0. \tag{4}$$

$$\Lambda = \pm\frac{3}{R^2}$$

where R – radius of curvature of space.

These spaces have global symmetry groups $SO(4,1)$ and $SO(3,2)$ that leave the metrics (3)-(4) invariant. Spaces (3)-(4) are called de Sitter worlds of the 1st and 2nd kind or de Sitter worlds DS and anti-de Sitter AdS.

We restrict ourselves to the de Sitter world (3) and the $SO(4,1)$ group. The case of the anti-de Sitter world will be considered in a separate paper, because of the difficulties in interpreting the space-time measurements.

The commutation relations for the generators of the group $SO(4,1)$ have the form:

$$[M_{\mu\nu}, M_{\varrho\sigma}] = i(g_{\mu\varrho}M_{\nu\sigma} - g_{\mu\sigma}M_{\nu\rho} - g_{\nu\varrho}M_{\mu\sigma} + g_{\nu\sigma}M_{\mu\varrho}),$$
$$[M_{\mu\nu}, P_\varrho] = i(g_{\mu\varrho}P_\nu - g_{\nu\varrho}P_\mu), \quad [P_\mu, P_\nu] = \frac{i}{R^2}M_{\mu\nu}, \quad P_\mu = \frac{1}{R}M_{4\mu}$$

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The Casimir operators of the Lie algebra of the group $SO(4,1)$ have the following form, [4]:

$$C_1 = \frac{1}{2R^2} M_{ab} M^{ab} = -P_\lambda P^\lambda - \frac{1}{2R^2} M_{\mu\nu} M^{\mu\nu} = M^2,$$

$$C_2 = -W_a W^a, \;\; W_a = \frac{1}{8R} \epsilon_{abcde} M^{bc} M^{de}$$

When $R \to \infty$ the Lie algebra of the group $SO(4,1)$ becomes over to the Lie algebra of the Poincarė group, and the Casimir operators become:

$$C_1 \to m^2, \quad C_2 \to m^2 s(s+1),$$

where m, s - spin and rest mass, respectively (this limiting transition corresponds to the well-known Wigner-Inönü contraction, [6]).

From the limiting transition of the Casimir operators C_1, C_2 it follows that unlike the Minkowski world in the de Sitter world, elementary systems are identified not by mass and spin, but by some functions of spin and mass.

In particular, non-degenerate representations of the group $SO(4,1)$ can be realized in the space of $2s+1$-component vector-functions and the degree of homogeneity of σ on the upper field of the cone, [7].

Then the parameters σ, s will play the role of invariants characterizing the irreducible representations of the group $SO(4,1)$. In addition, after the operation of contraction, the parameter s becomes into spin and the σ to the function of mass m.

The basis for our assumption about the nature of dark matter and energy is the following theorem on the contraction of representations of the de Sitter group $SO(4,1)$ to the representations of the Poincarė group $ISO(3,1)$.

Theorem. The result of the contraction of the UIR's $T^{(\sigma,s)}(g), g \in SO(4,1), \sigma = -3/2 + imR$, for $R \to \infty$ is the direct sum of UIR, $U^{(m,s,\varepsilon)}(g), g \in ISO(3,1)$ with mass σ, s and differing in energy sign ε.

Proof. The proof of the theorem is given in the article [8].

CONCLUSION

In the flat world of Minkowski, Wigner's elementary systems are determined by the rest mass, the spin and the sign of their energy can be identified with elementary particles. The considerations about the stability of physical systems require to limit the energy spectrum from below and to exclude negative energies.

In the de Sitter world, elementary Wigner systems are identified by spin and by a parameter, which is the flat limit of a function of spin and mass, with different energy signs. But unlike the Minkowski world, we cannot exclude negative energies from consideration.

That is, elementary systems on a cosmological scale can be in states with positive and negative energies. Elementary systems in a state with positive energy behave like a gravitating mass, and in a negative energy state as an anti-gravity mass.

And so, our final conclusions are as follows:

1. Mysterious "dark matter" and "dark energy" consist of such elementary systems.
2. "Dark matter" and "dark energy" are the first manifestations of quantum properties on the scale of the universe. Until now, quantum phenomena have been encountered in the micro-world, and also as macroscopic quantum effects in the theory of condensed matter.
3. "Dark matter" and "dark energy" are carriers of information about the first moments of the universe after the Big Bang.

The last conclusion follows from the fact that according to the standard cosmological model, the de Sitter world is a necessary phase of the evolution of the universe in the first instants ($10^{-34} - 10^{-32}$ seconds) after the Big Bang.

Of course, our universe is not de Sitter's world, although according to some data it is developing in the direction of this model. Considering the given phenomena of dark matter and dark energy in the general case is a difficult task because today there is no quantum theory of gravity. The solution of this problem in general for the gravitational field requires not only new physical concepts but also new mathematics. The main results of this work were reported in the 3rd International Scientific Conference "Modern Problems of Astrophysics-III", September 25-27, 2017, Georgia.

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