SPECTRAL DENSITY OF THE ULTRA SHORT LASER PULSES AT PARAMETRIC INTERACTION IN METAMATERIALS

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A three -wave parametric interaction in metamaterials is analyzed via consideration the negative refraction at the frequency of a signal wave. Analytic expression for the spectral density of a backward signal wave is obtained in the presence of group velocity mismatch and group velocity dispersion. When characteristic lengths of group velocity mismatch and the group velocity dispersion are less than the nonlinear length the excited pulse splits into narrow peaks . It is shown that, at the ratio of characteristic lengths $l_{nl}/l_{\nu} = 0$, the graph of the spectral density is symmetric relatively negative and positive values of phase modulation parameter.

Keywords: Metamaterials, parametric amplification, Gaussian pulse, second order dispersion.
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INTRODUCTION

The negative index materials (NIM) are attractive due to specifics of their interaction with electromagnetic waves [1,2]. Different signs of refractive index correspond to different frequency intervals of the interacting waves. Therefore the energy fluxes (Poynting vector) of the waves with a positive sign of refractive index will propagate in opposite direction to those with frequencies corresponding to a negative sign of refractive index. . The dynamics of three wave interaction in NIM was considered for the case of second harmonic generation in [3]. Results obtained in [4,5] are being used for the developments the metamaterials in the near IR and visible ranges of the spectrum. Earlier we have analyzed the efficiency of energy conversions between two direct waves with respect to the energy of the backward signal wave for the case of signal-wave amplification in metamaterials [6] in the constant intensity approximation (CIA) [7,8], taking into account the reverse reaction of excited wave on the exciting one. By employing the CIA we have studied the parametric interaction of optical waves in metamaterials under low-frequency pumping in the case of a negative index at a signal wave frequency [9] .The analytic expressions obtained in CIA showed , that the choice of the optimum parameters for the pump intensity, total length of the metamaterial and phase mismatch will facilitate obtaining the regimes of an effective amplification as well as the generation of signal wave. The characteristic processes observed at parametric interactions of running and counter waves in metamaterials are the transition processes [10]. Authors [11,12,13] were analyzed the transition processes by employment the first order dispersion theory in the medium with quadratic nonlinearity. In case of counter waves the phase matching condition is executed due to opposite directionality of the Poynting vector to the wave vector. To pump the nonlinear crystal of parametrical amplifier the nanosecond pulses of laser radiation are required. Parametric amplification of light in nonlinear crystals can be used for amplification the radiation being used with the aim of optical stochastic cooling of the relativistic heavy ions [14] Note that earlier we have employed constant intensity approximation to study the stationary optical parametric amplification [15] in the Fabri-Perrot cavity filled with dissipative dispersive nonlinear medium. Here optimization of various parameters such as the length of the nonlinear medium, wave mismatch, intensities of the pump and idler waves were considered to maximize the signal wave gain.

Under reduction in the pulse duration the character of interaction of modulated wave significantly depends on the dispersion properties of a medium. The frequency conversion for the ultra-short pulses with running wave was analyzed in [16]. Note that the growing interest to the non-stationary interaction of ultra-short pulses of light in nonlinear medium is related to the development of powerful sources of light pulses of femtosecond duration [1]. Earlier in [17] we were studied influence of group velocity mismatch (GVM) as well as group velocity dispersion (GVD) to the generation of sum frequency of ultra- short pulses in an external cavity under the phase matching and absence of linear losses . It was shown that in some cases efficiency of conversion in the existence of GVM and GVD can be significantly higher as compared as to the absence of mismatch and dispersion using the Gaussian pulse with quadratic phase modulation as the input pulse led to compression of spectrum with increase in GVM and decrease in GVD. It was obtained that maximum energy of conversion is reached not at group phase matching, but at the definite characteristic lengths of GVM and GVD.

2. DISCUSSIONS AND RESULTS

In the present work we investigate theoretically the non-stationary parametric amplification in metamaterials in the second order dispersion theory. We study this

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problem assuming that the nonlinear crystal has length l , and its cross section is much larger than the input laser beam. We ignore any reflections at the crystal surfaces. The beam axis (which we term) is normal to the crystal surface, and this is the direction of the input wave vector. The input surface of the crystal is at$z = l$. We assume for definiteness that for a parametric three-wave interaction in a metamaterials the medium is "left" at the frequency of the signal wave only. Here the pump wave is a long pump pulse with frequency ω_3 an idler wave is at frequency ω_2 and a signal wave is at the difference frequency $\omega_1 = \omega_3 - \omega_2$. The geometry of the problem is so that the pump and idler waves enter the nonlinear medium from the left $(z = 0)$, but the signal wave from the right $(z = l)$ hand side. In such a consideration the wave vectors of all interacting waves in a metamaterial propagate in the positive direction of the z axis. During the wave propagation in a nonlinear medium as a result of the nonlinear interaction the energy exchange occurs between the counter wave packets of two types : direct waves (the idler and pump waves) and an backward wave (the signal wave); this leads to the energy transfer from the pump and idler waves into the signal-wave energy. For the negative values of the dielectric permittivity and magnetic permeability at the signal wave frequency ω_1 and the positive values at the frequencies ω_2 , ω_3 the parametrical interaction is described by the system of parametrically coupled equations [1].

$$\left(\frac{\partial}{\partial z}+\frac{1}{u_1}\frac{\partial}{\partial t}-i\frac{g_1}{2}\frac{\partial^2}{\partial t^2}+\delta_1\right)A_1=-i\gamma_1A_3A_2^*e^{i\Delta z}$$

$$\left(\frac{\partial}{\partial z}+\frac{1}{u_2}\frac{\partial}{\partial t}-i\frac{g_2}{2}\frac{\partial^2}{\partial t^2}+\delta_2\right)A_2=-i\gamma_2A_3A_1^*e^{i\Delta z} \quad (1)$$

$$\left(\frac{\partial}{\partial z}+\frac{1}{u_3}\frac{\partial}{\partial t}-i\frac{g_3}{2}\frac{\partial^2}{\partial t^2}+\delta_1\right)A_3=-i\gamma_3A_1A_2e^{-i\Delta z}$$

here A_j (j=1-3) are the corresponding complex amplitudes of the signal, idler and pump waves respectively , δ_j are the absorption coefficients of the medium at frequencies ω_j (j=1-3), u_j are the group velocities of the interacting waves , $\Delta = k_1 - k_2 - k_3$ is the phase mismatch between the interacting waves , $g_j = \partial^2 k_j / \partial\omega_j^2$ (the 3-rd term in the Taylor expansion around the central frequency $\omega_0: \Delta\omega = \omega - \omega_0$, $k_n(\omega) \cong k_n(\omega_0) + k_n^{'}\Delta\omega + \frac{1}{2}k_n^{''}\Delta\omega^2 + \cdots$) is the dispersion of group velocities and γ_1 , γ_2 , γ_3, are the coefficients of nonlinear coupling

$$\gamma_1 == \frac{8\pi\chi_{eff}^2\,\omega_1^2|\varepsilon_1|}{k_1c^2}, \qquad \gamma_2 = \frac{8\pi\chi_{eff}^2\,\omega_2^2\varepsilon_2}{k_2c^2},$$

$$\gamma_3 = \frac{8\pi\chi_{eff}^2\,\omega_3^2\varepsilon_3}{k_3c^2},$$

where χ_{eff}^2 is the effective quadratic susceptibility of the medium.

Assuming pump wave amplitude to be constant ($A_3 = A_{30} = const.$) and having put substitution $\eta = t - \frac{z}{u_1}$ the set of above equations (1) is reduced to

$$\left(\frac{\partial}{\partial z}-i\frac{g_1}{2}\frac{\partial^2}{\partial\eta^2}+\delta_1\right)A_1(z,\eta)=-i\gamma_1A_{30}A_2^*(z,\eta)e^{i\Delta z}$$

$$\left(\frac{\partial}{\partial z}+\nu\frac{\partial}{\partial\eta}-i\frac{g_2}{2}\frac{\partial^2}{\partial\eta^2}+\delta_2\right)A_2(z,\eta)=$$
$$-i\gamma_2A_{30}A_1^*(z,\eta)e^{i\Delta z} \quad (2)$$

where $\nu = 1/u_2 - 1/u_1$ is a group velocity mismatch. To analyze the system (2) it is convenient to use the Fourier transformation

$$A_{1,2}(z,\eta) = \int_{-\infty}^{+\infty} A_{1,2}(z,\omega)e^{-i\omega\eta}d\omega \quad (3)$$

Substituting (3) into (2) yields

$$\left(\frac{\partial}{\partial z}+i\frac{g_1}{2}\omega^2+\delta_1\right)A_1(z,\omega)=-i\gamma_1A_{30}A_2^*(z,\omega)e^{i(\omega\eta+\Delta z)}$$
$$\left(\frac{\partial}{\partial z}+i\frac{g_2}{2}\omega^2-i\nu\omega+\delta_2\right)A_2(z,\omega)=-i\gamma_2A_{30}A_1^*(z,\omega)e^{i(\omega\eta+\Delta z)} \quad (4)$$

Solving this system in the absence of losses ($\delta_i = 0$) gives following expression for the amplitude of a signal wave

$$A_1(\omega,z) = \frac{i\gamma_1A_{30}A_2}{\lambda - k\tan\lambda\, l}(\cos\lambda z\cdot\tan\lambda l - \sin\lambda z)e^{-kz} \quad (7)$$

where $\lambda = l_{nl}^{\ -1}\ [\left(\frac{1}{4}\frac{l_{nl}}{l_{d.}}(\alpha+1)\omega^2\tau^2 - \frac{1}{2}\frac{l_{nl.}}{l_\nu}\omega\tau + \frac{\Delta}{\Gamma_3}\right)^2 - 1]^{1/2}$,
$k = l_{nl}^{\ -1}[i(\frac{1}{4}(\alpha-1)\frac{l_{nl}}{l_{d.}}\omega^2\tau^2 - \frac{l_{nl.}}{l_\nu}\omega\tau + \frac{\Delta}{\Gamma_3})\ ]$, $\alpha = \frac{g_2}{g_1}$, $l_d = \frac{\tau^2}{g_1}$, $l_\nu = \frac{\tau}{\nu}$

Furthermore we assume that the input wave is Gaussian with a quadratic phase modulation .

$$A_2(t) = A_{20}e^{-\frac{t^2}{2\tau^2} - i\gamma\frac{t^2}{2}} \quad (8)$$

Using the Fourier transformation

$$A_2(\omega) = \frac{A_{20}}{2\pi}\int_{-\infty}^{+\infty} e^{-\frac{t^2}{2\tau^2} - i\gamma\frac{t^2}{2}}e^{-i\omega t}dt \quad (9)$$

for the spectral density we obtain the following expression

$$S_2(\omega) = \frac{A_{20}\tau^2}{2\pi}\frac{1}{\sqrt{1+p}}e^{-\frac{\mu^2}{1+p}} \quad (10)$$

where $p = \gamma^2\tau^4$ and $\mu = \omega\tau$ are the frequency modulation and phase modulation parameters respectively. Substituting (9) into (7) for spectral density of a signal wave $S_1(\omega, z) = A_1(\omega, z) \cdot A_1^*(\omega, z)$ has resulted

$$S_1(\omega, z) = K\frac{e^{-\frac{\mu^2}{1+p}}(tan\lambda l \cdot cos\lambda z - sin\lambda z)^2}{(\lambda z)^2 + (kz)^2 tan^2\lambda l} \quad (11)$$

where $K = \frac{cn\gamma_1^2 I_{30} I_{20}\tau^2 z^2}{16\pi}$

From (11) it follows that the shape of a spectrum of a amplified signal wave is determined not only by the values of z, l_{nl}, l_v and l_d but also with their quotients z/l_{nl}, l_{nl}/l_v, l_{nl}/l_d. In Fig. 1 the dependences of a spectral density $S_1(\omega, z)$ on the phase modulation parameter $\omega\tau$ are illustrated at different values of l_{nl}/l_v and l_{nl}/l_d. As can be seen, the shape of a spectrum varies with the change in these ratios, in particular, when $l_{nl}/l_v = 0$ (curves 1 and 3), the spectrum becomes symmetric relatively negative and positive values of phase modulation parameter.

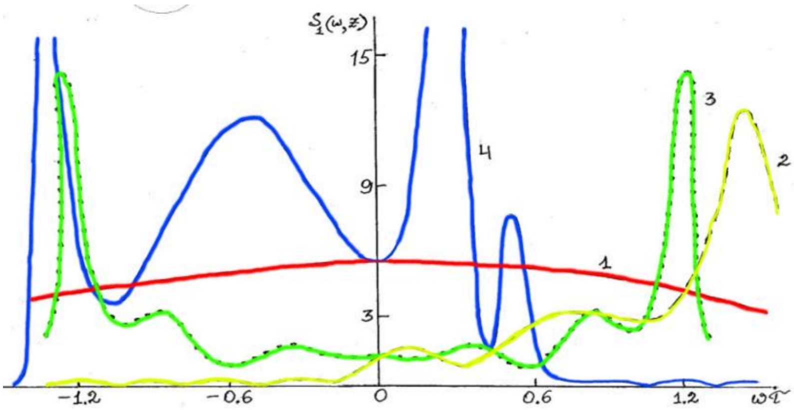

Fig. 1. The reduced spectral density $S_1(\omega, z)$ of a signal wave versus phase modulation parameter $\omega\tau$ for $p = 5$, $z/l_{nl} = 0{,}5$, $\Delta = 0$, $\delta_i = 0$: $1 - l_{nl}/l_v = l_{nl}/l_d = 0$; $2 - l_{nl}/l_v = 3$, $l_{nl}/l_d = 0$; $3 - l_{nl}/l_v = 0$, $l_{nl}/l_d = 3$; $4 - l_{nl}/l_v = l_{nl}/l_d = 3$

In Fig. 2 a spectral density is given as a function of phase modulation at different values of intensity of idler wave. As can be seen at the same values of input intensity (curves 1 and 2) increase in frequency modulation leads to increase in spectral density, however at equal frequency modulations increase in intensity decreases the spectral density of an amplified signal wave (curves 1 and 3).

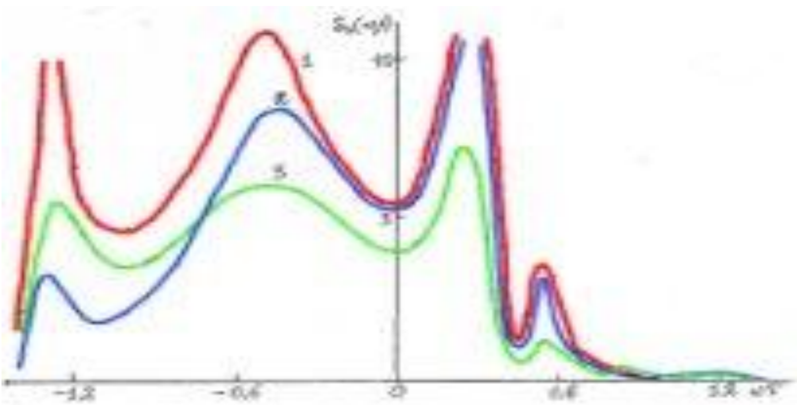

Fig. 2. The reduced spectral density $S_1(\omega, z)$ of a signal wave as a function of phase modulation parameter $\omega\tau$ for $z/l_{nl} = 0{,}7$ (curves 1 and 2) and $z/l_{nl} = 1$ (curve 3), $p = 0$(curve 2), $p = 5$ (curves 1 and 3) and $\Delta = 0$, $\delta_i = 0$

Effect of phase modulation of idler wave onto the spectral density of amplified signal wave also is demonstrated in Fig .3.

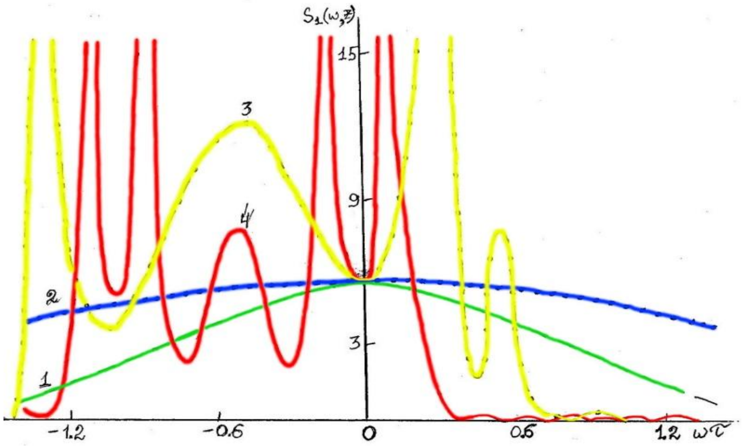

Fig. 3. Dependences of a spectral density $S_1(\omega,z)$ of a signal wave on the phase modulation parameter $\omega\tau$ for $p=0$ (curve 1), $p=5$ (curves $2-4$) and z / $l_{nl}=0{,}5$, Δ= 0, $\delta_i=0$ at different values of ratios of characteristic lengths: $1-l_{nl}/l_v$ = $l_{nl}/l_d=0$; $2-l_{nl}/l_v$ = $l_{nl}/l_d=0$; $3-l_{nl}/l_v$ = $l_{nl}/l_d=3$; $4-l_{nl}/l_v$ = $l_{nl}/l_d=10$;

From Fig. 3 it is seen that, a spectrum is symmetric when $l_{nl}/l_v=0$ independently on the value of l_{nl}/l_d . Existence of phase modulation leads to increase in the width of spectrum of a signal wave. At larger values of a frequency modulation ($\gamma\tau^2\gg 1$) the splitting up occurs in the spectral density of amplified pulse (curve 4) . All curves in Fig. 1-3 are plotted for the same signs of the coefficients of group velocity dispersions. Note that when $g_1=g_2$, amplification of signal wave occurs without dispersion of group velocities. The graphs are plotted for the case when $g_2/g_1=3$.

CONCLUSION

From above mentioned one can conclude that parametric amplification of ultra-short pulses in metamaterial in the second order dispersion theory is affected by the influence of group velocity mismatch as well as the group velocity delay .

Here an analytical expressions for the spectral density of a signal wave was derived . We showed that the spectral density of a ultra-short pulse wave is affected by the ratios of characteristic lengths. When $l_{nl}/l_v=0$, the shape of a graph of the spectral density becomes symmetric relatively negative and positive values of phase modulation parameters and has a maximum at positive values of phase modulation when $l_{nl}/l_d=0$.

For the ratios of characteristic lengths differ from zero maxima of spectral density are obtained not at zero $\omega\tau$ but at different values of this parameter.

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