MAGNETIC MOMENT OF ELECTRONS IN DILUTED MAGNETIC SEMICONDUCTOR QUANTUM RING

A.M. BABANLI[1,2], B.G. IBRAGIMOV[2,3]
[1]*Department of Physics, University of Süleyman Demirel, 32260 Isparta, Turkey,*
[2]*G.M. Abdullayev Institute of Physics of Azerbaijan NAS, 131, H. Javid ave., Baku, AZ 1143,*
[3]*Baku State University, Baku, Azerbaijan*
arifbabanli@sdu.edu.tr, behbud.ibrahimov.93@mail.ru

In the present paper, we have investigated the magnetization of electrons in a diluted magnetic semiconductor (DMS) quantum ring. We take into account the effect of Rashba spin-orbit interaction, the exchange interaction and the Zeeman term on the magnetization. We have calculated the energy spectrum and wave function of the electrons in diluted magnetic semiconductor quantum ring. Moreover, we have calculated the magnetic moment as a function of the magnetic field for strong degenerate electron gas at finite temperature of a diluted magnetic semiconductor quantum ring.

Keywords: Rashba effect, magnetic moment, heat capacity, diluted magnetic semiconductor.
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1. INTRODUCTION

In the last decade enormous attention has been devoted toward control and engineering of spin degree of freedom at mesoscopic scale, usually referred to as spintronics [3]. Important class of materials for spintronics forms diluted magnetic semiconductors (DMS). In a previous paper [1] we calculated the heat capacity and magnetization of a DMS quantum ring for Boltzmann statistics. The aim of this paper is to generalize the theory of free-electron Landau diamagnetism so as to include parabolic of the Fock-Darwin type confinement. In this way we move from classical statistics to the degenerate Fermi limit.

2. THEORY

We take into account the effects of the Zeeman and exchange terms on the magnetic moment of DMS quantum ring, the electron is assumed to be moving in a parabolic potential of the Fock -Darwin type given by [1]:

$$V_c(\rho) = \frac{V_0\rho^2}{2R^2},\ \rho \le R, \tag{1}$$

where V_0 - defines the depth of this potential and $\rho -$ is the distance of electron from the centre of the DMS quantum ring. The quantum ring is subjected to a uniform magnetic field $\vec{H} = (0,0,H)$ normal to the quantum ring plane. We assume that the spin-orbit interaction is described by the Rashba Hamiltonians [1]. The total Hamiltonian of the system is given by:

$$H = \frac{1}{2m_n}\left(\vec{P} + e\vec{A}\right)^2 + V_c(\rho) + \frac{1}{2}g\sigma_z\mu_B H + \sigma_z\alpha_0\frac{dV(\rho)}{d\rho}\left(-i\frac{1}{\rho}\frac{\partial}{\partial\varphi} + \frac{eH\rho}{2\hbar}\right) + H_{ex}, \tag{2}$$

where $m_n -$ is the electron mass, $\sigma_z -$ is the Pauli z matrix, α_0 is the Rashba spin-orbit coupling parameter, g represents the Lande factor. In the mean field approximation, the exchange Hamiltonian term can be written as:

$$H_{ex} = \frac{1}{2}\langle S_z\rangle N_0 x J_{sd}\sigma_z = 3A\sigma_z \tag{3}$$

where J_{sd} is a constant which describes the exchange interaction; N_0 is the density of the unit cell. For uniform magnetic field, H directed along z-axis, the vector potentials in cylindrical polar coordinates have the components $A_\phi = \frac{H_\rho}{2}, A_\rho = 0$ and the solution of Schrödinger equation has been known [1]. The electron energy levels given by [1, 2]:

$$E_{nl\sigma} = \hbar\Omega_\sigma\left(n + \frac{1}{2} + \frac{|l|}{2}\right) + \frac{l\cdot\hbar\omega_c}{2} + \frac{1}{2}g\sigma_z\mu_B H + 3A\sigma + \sigma\frac{\alpha\cdot l}{R} \tag{4}$$

131 H.Javid ave, AZ-1143, Baku
ANAS, G.M.Abdulayev Institute of Physics
E-mail: jophphysics@gmail.com

where $\sigma = \pm 1$ and we have used notations:

$$\Omega_\sigma = \sqrt{4\omega_0^2 + \omega_c^2 + \sigma\frac{\alpha \cdot \omega_c}{R \cdot \hbar}},\ \ \omega_0 = \sqrt{\frac{V_0}{m_n R^2}},\ \ \omega_c = \frac{eH}{m_n} \tag{5}$$

The partition function for the Boltzmann statistics is given by:

$$z = \sum_\sigma \frac{1}{2}\frac{e^{-\frac{\sigma d}{k_B T}}}{cosh\left(\frac{\hbar\Omega_\sigma}{2k_B T}\right) - cosh\left(\frac{b_\sigma}{2k_B T}\right)};\ b_\sigma = \hbar\omega_c\left(1 + \sigma\frac{2\alpha}{\hbar\omega_c R}\right),\ d = \frac{1}{2}g\mu_B H + 3A, \tag{6}$$

where $E_{nl\sigma}$-is the energy spectrum of considered system, k_B is the Boltzmann constant. To calculate thermodynamic potential Ω we use an approach based on calculating the classical partition function z of the electron gas:

$$\Omega = -k_B T\frac{1}{2\pi i}\int_{\nu - i\infty}^{\nu + i\infty}\frac{\pi\xi}{\sin(\pi\xi)} \cdot \frac{e^{\frac{\mu}{k_B T}\xi}}{\xi^2} z\left(\frac{k_B T}{\xi}\right) d\xi \tag{7}$$

where μ is the chemical potential of the gas. If we change to the dimensionless variable of integration $z = \frac{b_\sigma}{2k_B T}\xi$, Eq.(7) takes the form

$$\Omega_\sigma = -\frac{b_\sigma}{4}\frac{1}{2\pi i}\int_{\nu - i\infty}^{\nu + i\infty}\frac{\pi\frac{2k_B T}{b_\sigma}z}{\sin\left(\pi\frac{2k_B T}{b_\sigma}z\right)} \cdot \frac{e\frac{2\mu}{b_\sigma}}{z^2}\frac{e - \frac{2\mu}{b_\sigma}}{\cosh(B_\sigma z) - \cosh(z)} dz, \tag{8}$$

where $B_\sigma = \frac{\hbar\Omega_\sigma}{b_\sigma}$. The finite temperature effects are represented by an expansion the functions $\frac{\pi\frac{2k_B T}{b_\sigma}z}{\sin\left(\pi\frac{2k_B T}{b_\sigma}z\right)}$ in powers of the small quantity $\pi\frac{2k_B T}{b_\sigma}z$.

$$\frac{\pi\frac{2k_B T}{b_\sigma}z}{\sin\left(\pi\frac{2k_B T}{b_\sigma}z\right)} \approx 1 + \frac{2\pi^2 k_B^2 T^2}{3b_\sigma^2} + \ldots \tag{9}$$

For the low fields $\frac{\mu}{\hbar\omega_c} >> 1$, only the small z behaviour of the non-exponential portion of the integrand in (8) contributes significantly:

$$\frac{1}{\cosh(B_\sigma z)-\cosh(z)} \approx \frac{1}{z^2}\frac{2}{B_\sigma^2-1}+\frac{1}{6}\frac{1+B_\sigma^2}{1-B_\sigma^2}+\ldots \tag{10}$$

Inserting (9) and (10) into (8) and using the formula

$$\frac{1}{2\pi i}\int_{v-i\infty}^{v+i\infty}\frac{dz}{z^\delta}e^{tz}=\frac{t^{\delta-1}}{\Gamma(\delta)}, \tag{11}$$

When δ positive, we find

$$\Omega=-\frac{1}{4}\sum_{\sigma=\pm1} b_\sigma\left(\frac{1}{2}\frac{D_\sigma^3}{B_\sigma^2-1}+\frac{D_\sigma^2}{6}\frac{1+B_\sigma^2}{1-B_\sigma^2}+\frac{4k_B^2T^2}{3b_\sigma^2\left(B_\sigma^2-1\right)}D_\sigma\right), \tag{12}$$

where $D_\sigma=\dfrac{2\mu-\sigma d}{b_\sigma}$. In the absence of spin degree of freedom the thermodynamic potential

$$\Omega=-\frac{1}{6}\frac{\mu^3}{\eta^2\omega_0^2}+\frac{\mu}{6}\left(1+\frac{\omega_c^2}{2\omega_0^2}-\frac{2k_B^2T^2}{\eta^2\omega_0^2}\right) \tag{13}$$

The magnetic moment of electrons in the quantum ring at the chemical potential $\mu=const$ is

$$M=-\frac{1}{A}\left(\frac{\partial\Omega}{\partial H}\right)_\mu=-\frac{2}{3}\frac{H}{A}\frac{\mu\mu_B^2}{\eta^2\omega_0^2}\left(\frac{m_0}{m_n}\right)^2, \tag{14}$$

where A is the area of quantum ring, m_0 is the free electron mass. We shall take as the area of the cross-section of the potential well where $\mu=\dfrac{m_n\omega_0^2r^2}{2}$ and can be written $A=\dfrac{2\pi\mu}{m_n\omega_0^2}$.

$$\frac{M}{\mu_B}=-\frac{m_n}{3\pi}\frac{\mu_B H}{\eta^2}\left(\frac{m_0}{m_n}\right)^2 \tag{15}$$

3. RESULTS AND DISCUSSION

Thus, the magnetization is independent of the confinement parameter ω_0.
We next turn to de Haas–van Alphen oscillatory behaviour the magnetization in quantum ring with Rashba spin-orbit coupling. The integrand in (8) has simple poles and the points $2il\pi/(B_\sigma\pm1)$, $l=\pm1,\pm2,\ldots$ along the imaginary axis. Evaluating (8) by closing the integration by a large semicircle to the left, and summing the residues we have:

$$\Omega=k_BT\sum_{\sigma=\pm1,l=1}\frac{(-1)^{l+1}}{2l}\left[\frac{sin\left(\frac{\mu-\sigma d4l\pi}{B_\sigma+1\, b_\sigma}\right)}{sinh\left(\frac{4l\pi}{B_\sigma+1}\frac{\pi k_BT}{b_\sigma}\right)sin\left(\frac{B_\sigma-1}{B_\sigma+1}l\pi\right)}+\frac{sin\left(\frac{\mu-\sigma d4l\pi}{B_\sigma-1\, b_\sigma}\right)}{sinh\left(\frac{4l\pi}{B_\sigma-1}\frac{\pi k_BT}{b_\sigma}\right)sin\left(\frac{B_\sigma+1}{B_\sigma-1}l\pi\right)}\right] \tag{16}$$

The structure of the resulting oscillations is quite complex since B_σ is strongly field dependent, and includes spikes where $\frac{B_\sigma - 1}{B_\sigma + 1}$ possesses integer values. At very high fields, so B_σ approaches 1 the amplitude the first term in (16) will grow without as expected. Differentiating only the rapidly oscillating factors in Eq. (16) we find the magnetization.

$$\frac{M}{\mu_B} = -\frac{k_B T m_0}{m_n} \sum\nolimits_{\sigma=\pm 1, l=1} \frac{(-1)^{l+1}}{2l} \left[\frac{cos\left(\frac{\mu - \sigma d 4 l\pi}{B_\sigma + 1\, b_\sigma}\right) \partial_{\hbar\omega_c}\left(\frac{\mu - \sigma d 4 l\pi}{B_\sigma + 1\, b_\sigma}\right)}{sinh\left(\frac{4l\pi}{B_\sigma + 1}\frac{\pi k_B T}{b_\sigma}\right) sin\left(\frac{B_\sigma - 1}{B_\sigma + 1} l\pi\right)} + \frac{cos\left(\frac{\mu - \sigma d 4 l\pi}{B_\sigma - 1\, b_\sigma}\right) \partial_{\hbar\omega_c}\left(\frac{\mu - \sigma d 4 l\pi}{B_\sigma - 1\, b_\sigma}\right)}{sinh\left(\frac{4l\pi}{B_\sigma - 1}\frac{\pi k_B T}{b_\sigma}\right) sin\left(\frac{B_\sigma + 1}{B_\sigma - 1} l\pi\right)} \right] \quad (17)$$

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