POLARIZATION EFFECTS AT HIGGS BOSON DECAY $H \Rightarrow f\bar{f}\gamma$

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In the framework of the Standard Model, the process of the radiation decay of Higgs boson into a fermion-antifermion pair was studied: $H \Rightarrow f\bar{f}\gamma$. Taking into account the spiralities of fermions and circular polarization of the γ -quanta an analytical expression is obtained for the decay width. The mechanisms of bremsstrahlung of a photon by a fermion pair, as well as fermion and W-boson loop diagrams, are considered in detail. The circular polarization of the γ -quanta was studied depending on the angle θ and the invariant mass x of the fermion pair.

Keywords: Standard Model, Higgs boson, fermion pair, circular polarization, decay width.
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1. INTRODUCTION

The standard model (SM), based on the local gauge symmetry $SU_C(3) \times SU_L(2) \times U_Y(1)$, satisfactorily describes the strong and electroweak interactions of quarks, leptons, and gauge bosons [1, 2]. A doublet of scalar complex fields $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^o \end{pmatrix}$ is introduced into the theory, the neutral component of which has a non-zero vacuum value. As a result, the electroweak group $SU_L(2) \times U_Y(1)$ spontaneously breaks down to the electromagnetic group $U_Q(1)$. In this case, three of the four components of the scalar field are absorbed by gauge bosons. The lagging fourth component of the scalar field is Higgs boson H. The standard Higgs boson was discovered by the ATLAS and CMS collaborations in 2012 at CERN at the Large Hadron Collider (LHC) [3, 4] (see reviews [5–7]) and this began a new stage of research to elucidate the nature of Higgs boson.

The standard Higgs boson can decay through different channels (see [1, 8]). One of the main channels of Higgs boson decay is the decay $H \Rightarrow \gamma + \gamma$, $H \Rightarrow \gamma + Z$ which was studied in [1, 8-10]. Along with these decay channels, much attention is also paid to the radiative decay $H \Longrightarrow f + \bar{f} + \gamma$, where $f\bar{f}$ is a pair of fundamental fermions (leptons, quarks) [11-15]. In these works, the decay width $H \Longrightarrow f + \bar{f} + \gamma$ was determined, the distribution of the fermion pair over the invariant mass, also the angular asymmetry of the front and back and the degrees of longitudinal and transverse polarizations of the fermions were studied. However, the circular polarization of the γ-quantum is not considered in these works.

The aim of this work is to study the circular polarization of a γ -quanta in decay

$$H \Longrightarrow f + \bar{f} + \gamma, \tag{1}$$

where $f\bar{f}$ is the fermion pair (lepton $\tau^-\tau^+$or quark $c\bar{c}, b\bar{b}$, pair). In the framework of the SM, taking into account the longitudinal polarizations of the fermion pair and the circular polarization of the photon, an analytical expression is obtained for the decay width. The dependence of the degree of circular polarization of a photon on the invariant mass of a fermion pair is studied in detail.

2. THE RADIATION OF PHOTON BY A FERMION PAIR

The radiation decay of the standard Higgs boson into a fermion pair is described by two types of Feynman diagrams which are shown in fig. 1. Diagrams a) and b) correspond to the bremsstrahlung of a photon by a fermion pair, and diagrams c), d), e), f) and g) are fermion and W- boson loop diagrams.

The amplitude corresponding to diagrams a) and b) of fig. 1 can be written as follows:

$$M_{i\to f} = i g_{Hff} e Q_f [\bar{u}_f(p_1, \lambda_1) R \upsilon_f(p_2, \lambda_2)], \tag{2}$$

where

$$R = \hat{e}^* \cdot \frac{\hat{p}_1 + \hat{k} + m_f}{(p_1+k)^2 - m_f^2} - \frac{\hat{p}_2 + \hat{k} - m_f}{(p_2+k)^2 - m_f^2} \cdot \hat{e}^*,$$

$g_{Hff} = m_f[\sqrt{2}G_F]^{1/2}$is Higgs boson coupling constant with the fermion pair, m_f and Q_f is mass and charge of the fermion f, e^* is 4-polarization vector of photon, p, p_1, p_2 and k are 4-momenta of Higgs boson, fermion, antifermion and photon, respectively, λ_1 and λ_2 are spiralities of fermion and antifermion.

Applying the Dirac equations $\bar{u}_f(p_1,\lambda_1)(\hat{p}_1 - m_f) = 0, (\hat{p}_2 + m_f)\upsilon_f(p_2,\lambda_2) = 0$, the amplitude (2) can be changed to:

$$M_{i\to f} = iA_0[\bar{u}_f(p_1,\lambda_1) R \upsilon_f(p_2,\lambda_2)]. \tag{3}$$

Here

$$A_0 = -\frac{2\pi\alpha_{KED} m_f}{M_W sin\theta_W}, \tag{4}$$

$$R = \frac{2(e^*\cdot p_1) + \hat{e}^*\hat{k}}{2(p_1\cdot k)} - \frac{2(e^*\cdot p_2) + \hat{k}\hat{e}^*}{2(p_2\cdot k)},$$

M_W is the mass of W -boson, θ_W is Weinberg angle.

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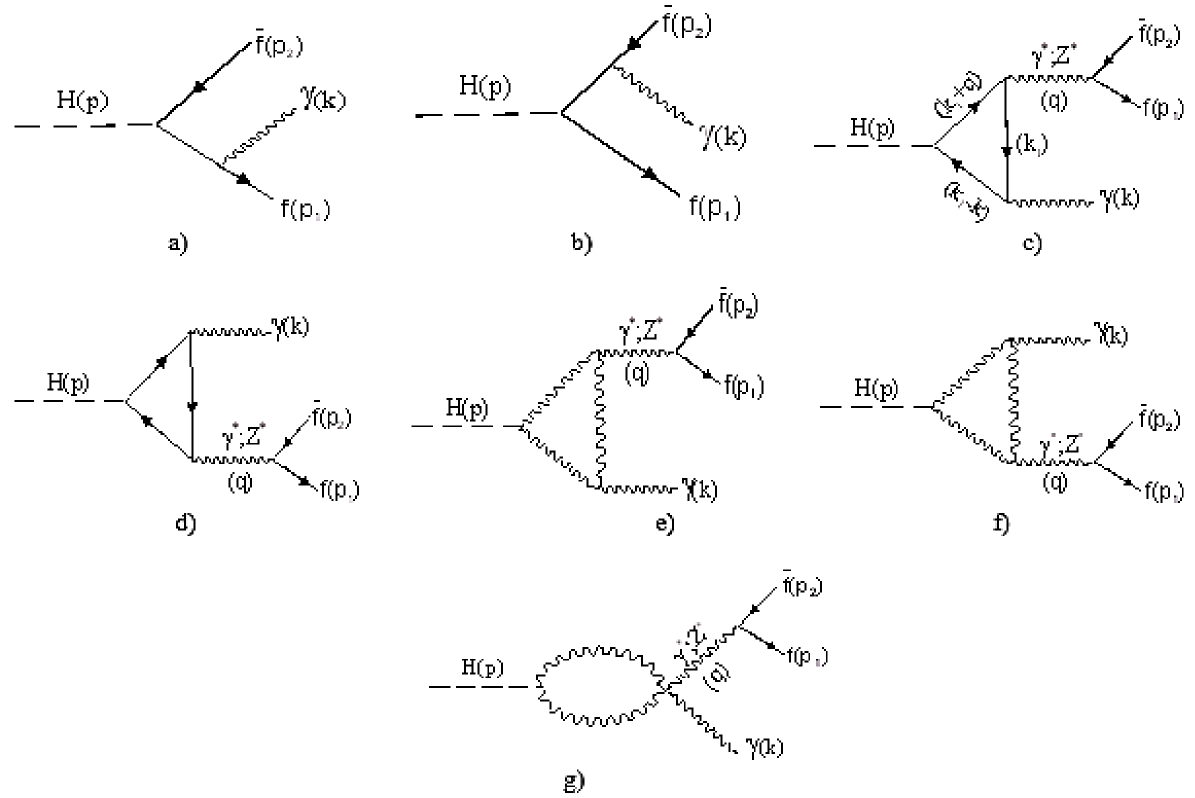

Fig. 1. Feynman diagrams for decay $H \Rightarrow f\bar{f}\gamma$.

The constant of interaction g_{Hff} increases when the mass of the fermion pair increases. Therefore, the decay of the standard Higgs boson with a mass of 125 GeV can produce fermion pairs $\tau^-\tau^+$, $c\bar{c}$ and $b\bar{b}$. Due to the small mass m_f, the decay channels $H \Rightarrow e^- + e^+ + \gamma$,$H \Rightarrow \mu^- + \mu^+ + \gamma$,$H \Rightarrow u + \bar{u} + \gamma$, $H \Rightarrow d + \bar{d} + \gamma$ and $H \Rightarrow s + \bar{s} + \gamma$ are suppressed. The study of the radiative decay of $H \Rightarrow \tau^- + \tau^+ + \gamma$ shows particular interest, since the decay channels of $\tau^- \Rightarrow \pi^- + \nu_\tau$, $\tau^- \Rightarrow K^- + \nu_\tau$, $\tau^- \Rightarrow \rho^- + \nu_\tau$, decay make it possible to measure the polarization of the τ - lepton. In addition, in the decay of $H \Rightarrow \tau^- + \tau^+ + \gamma$, a photon can acquire circular polarization, the measurement of which is a source of additional information about the standard Higgs boson.

Note that in the radiation decays of Higgs boson $\Rightarrow \tau^- + \tau^+ + \gamma$, $H \Rightarrow c + \bar{c} + \gamma$ and $H \Rightarrow b + \bar{b} + \gamma$ the ratios are $\frac{m_f^2}{M_H^2} = 0{,}0002 \ll 1$, $\frac{m_c^2}{M_H^2} = 0{,}00017 \ll 1$, and $\frac{m_b^2}{M_H^2} = 0{,}00015 \ll 1$. Therefore, we can neglect the terms, which are proportional to $\frac{m_f^2}{M_H^2}$. In this case, on the basis of amplitude (3) for the decay width $H \Rightarrow f + \bar{f} + \gamma$, the following expression is obtained (in the system of the center of mass of the fermion pair $\vec{p}_1 + \vec{p}_2 = \vec{q} = 0$):

$$\frac{d\Gamma}{dxdz} = \frac{A_0^2 M_H v}{2^{10}\pi^3(1-x)} \cdot \frac{N_C}{(1-v^2z^2)^2} \times \{(1+\lambda_1\lambda_2)(1+x^2)(1-v^2z^2) + +s_\gamma(\lambda_1+\lambda_2)(1-x)[2xv^2(1-z^2)+(1-x)(1-v^2z^2)]\}. \qquad (5)$$

Here $s_\gamma = \pm 1$, it characterizes the circular polarization of the photon (for $s_\gamma = +1$ the photon has right circular polarization, and for $s_\gamma = -1$ has the left one), $z = cos\theta, \theta$ is the angle between the directions of Higgs boson and fermion momenta, x determines the invariant mass of the fermion pair in units of M_H^2:

$$x = \frac{q^2}{M_H^2} = \frac{s}{M_H^2} = \frac{(p_1+p_2)^2}{M_H^2},$$

$v = \sqrt{1 - \frac{4m_f^2}{s}}$ is helicity of fermion, N_C is color factor (for the lepton pair $N_C = 1$, and for the quark pair $N_C = 3$).

From the decay width (5) of $H \Rightarrow f + \bar{f} + \gamma$ it follows that the fermion and antifermion must have the same spiralities: $\lambda_1 = \lambda_2 = \pm 1$ ($f_L\bar{f}_L$ or $f_R\bar{f}_R$, where f_L and f_R are the left and right fermions). This is due to the conservation of the total moment in the $H \Rightarrow f + \bar{f}$ transition. The decay width (5) also shows that when a longitudinally polarized fermion pair is produced, the emitted photon acquires circular polarization.

We determine the degree of circular polarization of the γ - quanta in the standard way:

$$P_\gamma = \frac{d\Gamma(\lambda_1; s_\gamma=+1)/dxdz - d\Gamma(\lambda_1; s_\gamma=-1)/dxdz}{d\Gamma(\lambda_1; s_\gamma=+1)/dxdz + d\Gamma(\lambda_1; s_\gamma=-1)/dxdz} = \lambda_1 \cdot \frac{(1-x)[2xv^2(1-z^2)+(1-x)(1-v^2z^2)]}{(1+x^2)(1-v^2z^2)} \quad (6)$$

Figure 2 shows the angular dependence of the degree of circular polarization of the photon for various invariant masses of the $\tau^-\tau^+$ lepton pair: $x = 0{,}102$; $x = 0{,}23$ and $x = 0{,}518$. It can be seen that for the values of the cosines of the angle $0 \le z \le 0{,}9$, the degree of circular polarization of the photon in the $H \Rightarrow \tau^- + \tau^+ + \gamma$ process is almost constant, and for $0{,}9 < z \le 1$, with increasing cosines of the angle θ, the degree of circular polarization of the photon decreases. An increase in the energy x carried away by the $\tau^-\tau^+$ lepton pair leads to a decrease in the degree of circular polarization of the photon.

Figure 3 illustrates the dependence of the degree of circular polarization of a photon in the $H \Rightarrow \tau^- + \tau^+ + \gamma$ decay on the invariant mass x at $z = 0$ and $z = 1$. As can be seen, with an increase in the energy x carried away by the lepton pair $\tau^-\tau^+$, the degree of circular polarization of the photon monotonically decreases and vanishes at the end of the spectra (for $x = 1$).

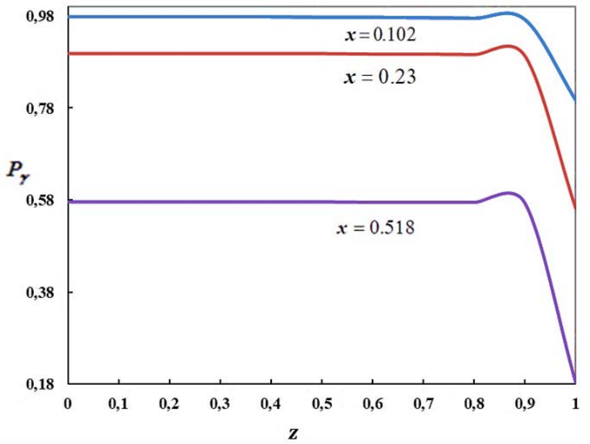

Fig. 2. Angular dependence of the degree of circular polarization of a photon in the $H \Rightarrow \tau^-\tau^+\gamma$ decay at $\lambda_1 = +1$.

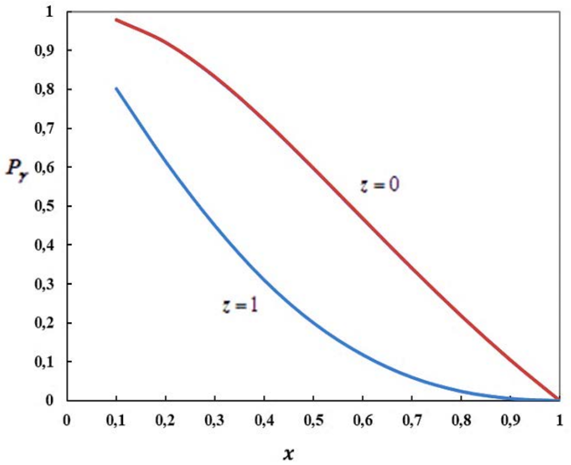

Fig. 3. Dependence of the degree of circular polarization of a photon in the $H \Rightarrow \tau^-\tau^+\gamma$ decay on the invariant mass x for $\lambda_1 = +1$.

The decay width $H \Rightarrow f + \bar{f} + \gamma$, summed over the polarization states of the photon and fermion pair, is given by:

$$\frac{d\Gamma}{dxdz} = \frac{A_0^2 M_H N_C v}{128\pi^3(1-v^2z^2)} \cdot \frac{1+x^2}{1-x}. \quad (7)$$

In fig. 4 shows the dependence of the decay width $H \Rightarrow \tau^- + \tau^+ + \gamma$ on the invariant mass x at $M_H = 125$ GeV, $m_\tau = 1.778$ GeV, $M_W = 80.385$ GeV, $sin^2\theta_W = 0.2315$ and various values of the cosines of the angle θ: $z = 0$; $z = 0.5$. As can be seen, with an increase in the invariant mass x, the decay width $H \Rightarrow \tau^- + \tau^+ + \gamma$ increases, a decrease in the exit angle θ leads to an increase in the decay width.

Integrating (5) over the particle exit angles θ, for the decay width $H \Rightarrow f + \bar{f} + \gamma$ we find the following expression that determines the distribution of the fermion pair over the invariant mass x:

$$\frac{d\Gamma}{dx} = \frac{A_0^2 M_H v}{2^{10}\pi^3} \cdot \frac{N_C}{1-x}\{(1+\lambda_1\lambda_2)(1+x^2)\cdot L + s_\gamma(\lambda_1+\lambda_2)(1-x)[-2x+(1+xv^2)L]\}, \quad (8)$$

where

$$L = \frac{1}{v} ln\frac{1+v}{1-v}.$$

The degree of circular polarization of the photon integrated over the particle angles θ is determined by the formula:

$$P_\gamma = \lambda_1 \cdot \frac{(1-x)[-2x+(1+xv^2)L]}{(1+x^2)L} \quad (9)$$

Figure 5 shows the energy dependence of the degree of circular polarization of the photon in the $H \Rightarrow \tau^- + \tau^+ + \gamma$ decay at $M_H = 125$ GeV, $m_\tau = 1.778$ GeV. With increasing x, the degree of circular polarization of the photon decreases.

Figure 5 shows the energy dependence of the degree of circular polarization of the photon in the $H \Rightarrow \tau^- + \tau^+ + \gamma$ decay at $M_H = 125$ GeV, $m_\tau = 1.778$ GeV. With increasing x, the degree of circular polarization of the photon decreases.

The decay width $H \Rightarrow f + \bar{f} + \gamma$, which characterizes the distribution of the fermion pair over the invariant mass x without taking into account the polarizations of the particles, is determined by the expression:

$$\frac{d\Gamma}{dx} = \frac{A_0^2 M_H N_C}{128\pi^3} \cdot \frac{1+x^2}{1-x} ln\left(\frac{1+v}{1-v}\right). \quad (10)$$

Figure 6 illustrates the dependence of the $H \Rightarrow \tau^- + \tau^+ + \gamma$ decay width on the invariant mass x at $M_H = 125$ GeV. With an increase in the fraction of energy carried away by the $\tau^-\tau^+$ lepton pair, the $H \Rightarrow \tau^- + \tau^+ + \gamma$ decay width increases.

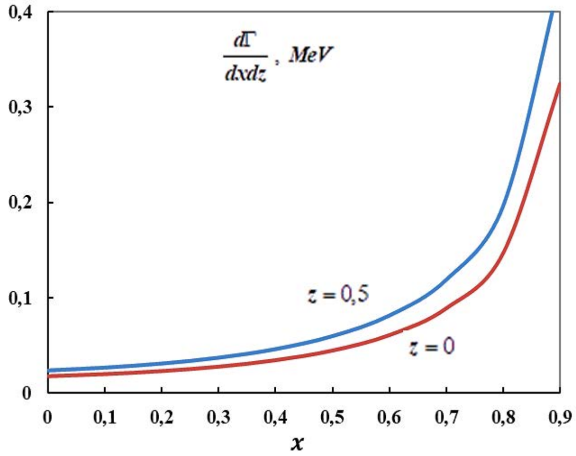

Fig. 4. Dependence of the decay width $H \Rightarrow \tau^-\tau^+\gamma$ on the invariant mass x .

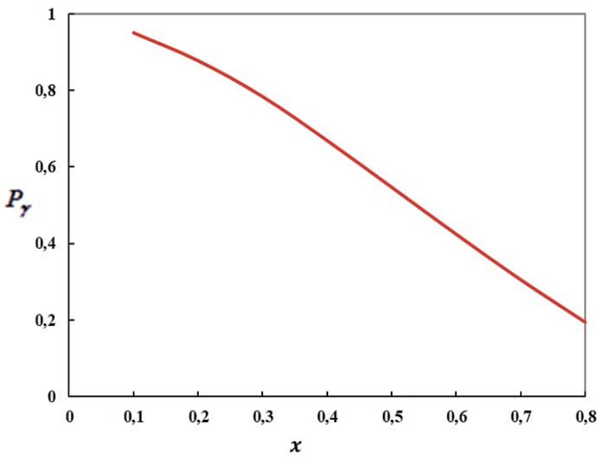

Fig. 5. The dependence of the circular polarization of the photon on x in the decay of $H \Rightarrow \tau^-\tau^+\gamma$ at $\lambda_1 = +1$

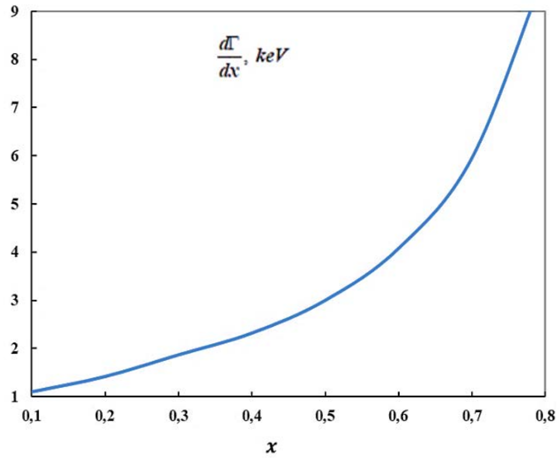

Fig. 6. Dependence of the $H \Rightarrow \tau^-\tau^+\gamma$ decay width on the invariant mass x

3. Fermion and W - boson loop diagrams

The amplitude corresponding to the bremsstrahlung of the photon by the fermion and antifermion (diagrams a) and b) in fig. 1) is proportional to the mass of the fermion m_f, therefore, the contribution of fermion and W -boson loop diagrams to Higgs decay of the $H \Rightarrow f + \bar{f} + \gamma$ boson can be significant. Typical single-loop Feynman diagrams are shown in fig. 1 (diagrams c), d), e), f) and g)). They are the photon and Z -boson pole diagrams of the $H \Rightarrow \gamma + \gamma^* \Rightarrow \gamma + f + \bar{f}$ and $H \Rightarrow \gamma + Z^* \Rightarrow \gamma + f + \bar{f}$ decays.

The single-loop Feynman diagrams shown in fig. 1 are crucial for the radiation production of a light fermion pair $H \Rightarrow e^- + e^+ + \gamma$, $H \Rightarrow u + \bar{u} + \gamma$, etc. We proceed to calculate the amplitude corresponding to these diagrams. The amplitude corresponding to fermion loop diagrams is written as:

$$M = ig_{Hff} \int \frac{d^4k_1}{(2\pi)^4} \cdot \frac{Sp[\gamma_\mu(\hat{k}_1+\hat{q}+m)(\hat{k}_1-\hat{k}+m)\gamma_\nu(\hat{k}_1+m)]}{(k_1^2-m^2)[(k_1+q)^2-m^2][(k_1-k)^2-m^2]} \cdot e_\nu^* \times \frac{g_{\mu\rho}}{q^2}[\bar{u}(p_1)\gamma_\rho \upsilon(p_2)], \tag{11}$$

where m is the mass of the loop fermion.

Using the Feynman integration technique, we can carry out integration over the 4 - momenta k_1, as a result, we have the amplitude:

$$M_{LOOP}^{(fermion)} = M_1 + M_2, \tag{12}$$

$$M_1 = (e^* \cdot q)\bar{u}(p_1,\lambda_1)[A_1\hat{k} + A_2\hat{k}\gamma_5]\upsilon(p_2,\lambda_2) - (k \cdot q)\bar{u}(p_1,\lambda_1)[A_1\hat{e}^* + A_2\hat{e}^*\gamma_5]\upsilon(p_2,\lambda_2),$$

$$M_2 = -i(\mu e^* kq)_\varepsilon \bar{u}(p_1,\lambda_1)[A_3\gamma_\mu + A_4\gamma_\mu\gamma_5]\upsilon(p_2,\lambda_2), \tag{13}$$

where

$$A_1 = g_V(f)D_Z(s)P_{s\gamma z} - \frac{1}{s}P_{\gamma\gamma} \ , \ A_2 = g_A(f)D_Z(s)P_{s\gamma z} \, ,$$

$$A_3 = g_V(f)D_Z(s)P_{a\gamma z} \ , \ A_4 = g_A(f)D_Z(s)P_{a\gamma z} \tag{14}$$

$$D_Z(s) = (s - M_Z^2 + iM_Z\Gamma_Z)^{-1}, \quad (\mu e^* kq)_\varepsilon = \varepsilon_{\mu\nu\rho\sigma} e_\nu^* k_\rho q_\sigma \ ,$$

$$g_V(f) = \frac{T_f - 2Q_f sin^2\theta_W}{2sin\theta_W \cdot cos\theta_W} \ , \ g_A(f) = \frac{T_f}{2sin\theta_W \cdot cos\theta_W} \ ,$$

$$P_{a\gamma z} = \frac{\alpha_{KED}^2}{M_W sin\theta_W} \cdot \frac{N_C e_f I_f}{sin\theta_W \ cos\theta_W} \ A_{f_2}(\tau_f \ , \lambda_f) \ ,$$

$$P_{s\gamma z} = \frac{\alpha_{KED}^2}{M_W sin\theta_W}\left(-2N_C \frac{I_f - 2e_f sin^2\theta_W}{sin\theta_W \ cos\theta_W} A_{f_1}(\tau_f \ , \lambda_f)\right) , \quad (15)$$

$$P_{\gamma\gamma} = \frac{\alpha_{KED}^2}{M_W sin\theta_W} \cdot (-4N_C) \cdot e_f^2 A_{f_1}(\tau_f \ , \lambda_f) \ ,$$

$$A_{f_1}(\tau \ , \lambda) = I_1(\tau, \lambda) - I_2(\tau, \lambda) \ ,$$

$$A_{f_2}(\tau \ , \lambda) = \frac{\tau\lambda}{\lambda - \tau}[2g(\tau) - 2g(\lambda) + f(\tau) - f(\lambda)], \quad (16)$$

$$I_1(\tau, \lambda) = \frac{\tau\lambda}{2(\tau - \lambda)} + \frac{\tau^2\lambda^2}{2(\tau - \lambda)^2}[f(\tau) - f(\lambda)] + \frac{\tau^2\lambda}{(\tau - \lambda)^2}[g(\tau) - g(\lambda)] \ ,$$

$$I_2(\tau, \lambda) = -\frac{\tau\lambda}{2(\tau - \lambda)}[f(\tau) - f(\lambda)] \ ,$$

and the functions $f(\tau)$ and $g(\tau)$ are equal:

$$f(\tau) = \begin{cases} arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4}\left[ln\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 & \tau < 1 \ , \end{cases}$$

$$(17)$$

$$g(\tau) = \begin{cases} \sqrt{\tau - 1} \ arcsin\frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ \frac{1}{2}\sqrt{1-\tau}\left[ln\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right] & \tau < 1 \ , \end{cases}$$

τ and λ are given by the relations:

$$\tau = \frac{4m^2}{M_H^2} \ , \ \lambda = \frac{4m^2}{s}.$$

Here e_f and I_f are the charge and the third projection of the weak isospin of the loop fermion; $g_V(f)$ and $g_A(f)$ are the vector and axial-vector coupling constants of fermion f; M_Z and Γ_Z are the mass and total decay width of the Z boson; T_f is the third projection of the weak isospin of fermion f, in the case of a t-quark loop we have: $\tau = \frac{4m^2}{M_H^2} > 1$, $\lambda = \frac{4m^2}{s} > 1$.

In the unitary gauge there are only three W-boson loop diagrams d), e) and f). Taking into account all the loop diagrams in Fig. 1, the decay amplitude of $H \Rightarrow f + \bar{f} + \gamma$ is determined by expression (12), but the expressions $P_{s\gamma z}$ and $P_{\gamma\gamma}$ change, they contain the contributions of both fermion and W - boson loop diagrams:

$$P_{s\gamma z} = \frac{\alpha_{KED}^2}{M_W sin\theta_W}\left[-ctg\theta_W A_W(\tau_W, \lambda_W) - 2N_C e_f \frac{I_f - 2e_f sin^2\theta_W}{sin\theta_W \ cos\theta_W} A_{f_1}(\tau_f \ , \lambda_f)\right] ,$$

$$P_{\gamma\gamma} = \frac{\alpha_{KED}^2}{M_W sin\theta_W} \cdot \left[-A_W(\tau_W \ , \ \lambda_W) - 4N_C e_f^2 A_{f_1}(\tau_f \ , \lambda_f)\right], \quad (18)$$

$$A_W(\tau, \lambda) = \left[\left(1 + \frac{2}{\tau}\right)\left(\frac{4}{\lambda} - 1\right) - \left(5 + \frac{2}{\tau}\right)\right] I_1(\tau, \lambda) + 16\left(1 - \frac{1}{\lambda}\right) I_2(\tau, \lambda).$$

The square of the $H \Rightarrow f + \bar{f} + \gamma$ decay amplitude, corresponding to fermion and W - boson loop diagrams, in the general case, has a complex structure and is given in the Appendix. However, in the center-of-mass system of a fermion pair ($\vec{q} = \vec{p}_1 + \vec{p}_2 = 0$), the square of Higgs boson decay amplitude is greatly simplified:

$$|M_{LOOP}|^2 = |M_1|^2 + |M_2|^2 + M_1^* M_2 + M_2^* M_1 \ , \quad (19)$$

$$|M_1|^2 = \frac{(M_H^2 - s)^2}{16} s\{(1 - \lambda_1\lambda_2)[(|A_1|^2 + |A_2|^2)(1 + v^2 z^2) + 4Re(A_1 A_2^*) s_\gamma v z] +$$
$$+ (\lambda_2 - \lambda_1)[(|A_1|^2 + |A_2|^2) \cdot 2s_\gamma v z + 2Re(A_1 A_2^*)(1 + v^2 z^2)]\} \ ,$$

$$|M_2|^2 = \frac{(M_H^2 - s)^2}{16} s\{(1-\lambda_1\lambda_2)[(|A_3|^2 + |A_4|^2)(1+v^2z^2) + 4Re(A_3A_4^*)s_\gamma vz] + +(\lambda_2-\lambda_1)[(|A_3|^2+|A_4|^2)\cdot 2s_\gamma vz + 2Re(A_3A_4^*)(1+v^2z^2)]\},$$

$$M_1^*M_2 + M_2^*M_1 = \frac{(M_H^2-s)^2}{16} s\{(1-\lambda_1\lambda_2)[2Re(A_1A_3^* + A_2A_4^*)s_\gamma(1+v^2z^2) + +4Re(A_1A_4^* + A_2A_3^*)vz] + (\lambda_2-\lambda_1)[4Re(A_1A_3^*+A_2A_4^*)vz + +2Re(A_1A_4^*+A_2A_3^*)s_\gamma(1+v^2z^2)]\}. \quad (20)$$

The differential decay width $H \Rightarrow f + \bar{f} + \gamma$, containing the contribution of fermion and W boson loop diagrams, can be written in the form:

$$\frac{d\Gamma}{dxdz} = \frac{(M_H^2-s)^3}{2^{11}\pi^3 M_H} sv\{(1-\lambda_1\lambda_2)[(|A_1|^2+|A_2|^2+|A_3|^2+|A_4|^2)(1+v^2z^2) + +4Re(A_1A_2^*+A_3A_4^*)s_\gamma vz + 2Re(A_1A_3^*+A_2A_4^*)s_\gamma(1+v^2z^2) + 4Re(A_1A_4^*+A_2A_3^*)vz] + +(\lambda_2-\lambda_1)[(|A_1|^2+|A_2|^2+|A_3|^2+|A_4|^2)2s_\gamma vz + 2Re(A_1A_2^*+A_3A_4^*)(1+v^2z^2) + +2Re(A_1A_4^*+A_2A_3^*)s_\gamma(1+v^2z^2) + 4Re(A_1A_3^*+A_2A_4^*)vz]\}, \quad (21)$$

It follows from the decay width (21) that the fermion and antifermion should have opposite spiralities: $\lambda_1 = -\lambda_2 = \pm 1$ ($f_R\bar{f}_L$ or $f_L\bar{f}_R$, where $f_L(\bar{f}_L)$ and $f_R(\bar{f}_R)$ are the right and left-polarized fermion (antifermion)). This is due to the preservation of the full moment in the transitions $\gamma^* \Rightarrow f + \bar{f}$ and $Z^* \Rightarrow f + \bar{f}$.

As noted in the previous section, when a photon is emitted by a fermion pair, the fermion and antifermion must have the same spiralities $\lambda_1 = \lambda_2 = \pm 1$ ($f_R\bar{f}_R$ or $f_L\bar{f}_L$). Thus, by the spiral properties of the fermion pair, we can separate the contribution of the loop diagrams to the decay width from the contribution of bremsstrahlung. At $\lambda_1 = -\lambda_2 = \pm 1$, the contribution to the decay amplitude of the $H \Rightarrow f + \bar{f} + \gamma$ diagrams of bremsstrahlung vanishes, and at $\lambda_1 = \lambda_2 = \pm 1$, on the contrary, the contribution of the loop diagrams vanishes.

Let us consider some particular cases of the decay width (21). We summarize the decay width according to the polarization states of the fermion pair:

$$\frac{d\Gamma}{dxdz} = \frac{(M_H^2-s)^3}{2^9\pi^3 M_H} sv\{[|A_1|^2+|A_2|^2+|A_3|^2+|A_4|^2](1+v^2z^2) + 4Re(A_1A_2^*+A_3A_4^*)s_\gamma vz + +4Re(A_1A_4^*+A_2A_3^*)vz + 2Re(A_1A_3^*+A_2A_4^*)s_\gamma(1+v^2z^2)\}. \quad (22)$$

Using the standard formula, we determine the degree of circular polarization of a photon in $H \Rightarrow f + \bar{f} + \gamma$ decay:

$$P_\gamma(s,z) = \frac{d\Gamma(s_\gamma=+1)/dxdz - d\Gamma(s_\gamma=-1)/dxdz}{d\Gamma(s_\gamma=+1)/dxdz + d\Gamma(s_\gamma=-1)/dxdz} = = \frac{2Re(A_1A_3^*+A_2A_4^*)(1+v^2z^2)+4Re(A_1A_2^*+A_3A_4^*)vz}{[|A_1|^2+|A_2|^2+|A_3|^2+|A_4|^2](1+v^2z^2)+4Re(A_1A_4^*+A_2A_3^*)vz}. \quad (23)$$

After summing the decay width (21) over the polarization states of the antifermion and photon, we have:

$$\frac{d\Gamma}{dxdz} = \frac{(M_H^2-s)^3}{2^9\pi^3 M_H} sv\{[|A_1|^2+|A_2|^2+|A_3|^2+|A_4|^2](1+v^2z^2) + 4Re(A_1A_4^*+A_2A_3^*)vz - -\lambda_1[2Re(A_1A_2^*+A_3A_4^*)(1+v^2z^2) + 4Re(A_1A_3^*+A_2A_4^*)vz]\}. \quad (24)$$

The degree of longitudinal polarization of the fermion is determined in the standard way:

$$P_f(s,z) = \frac{d\Gamma(\lambda_1=+1)/dxdz - d\Gamma(\lambda_1=-1)/dxdz}{d\Gamma(\lambda_1=+1)/dxdz + d\Gamma(\lambda_1=-1)/dxdz} = = -\frac{2Re(A_1A_2^*+A_3A_4^*)(1+v^2z^2)+4Re(A_1A_3^*+A_2A_4^*)vz}{[|A_1|^2+|A_2|^2+|A_3|^2+|A_4|^2](1+v^2z^2)+4Re(A_1A_4^*+A_2A_3^*)vz}. \quad (25)$$

The differential decay width $H \Rightarrow f + \bar{f} + \gamma$, containing the contribution of the loop diagrams, has the form:

$$\frac{d\Gamma}{dxdz} = \frac{(M_H^2 - s)^3}{2^8\pi^3 M_H} sv\{[|A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2](1 + v^2z^2) + 4Re(A_1A_4^* + A_2A_3^*)vz\} \quad (26)$$

Due to the second term proportional to z , angular asymmetry occurs back and forth, defined as

$$A_{FB}(s) = \frac{\int_0^1 \frac{d\Gamma}{dxdz}dz - \int_{-1}^0 \frac{d\Gamma}{dxdz}dz}{\int_0^1 \frac{d\Gamma}{dxdz}dz + \int_{-1}^0 \frac{d\Gamma}{dxdz}dz} = \frac{12Re(A_1A_4^* + A_2A_3^*)v}{[|A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2](3 + v^2)}. \quad (27)$$

Integrating the decay width (21) over the polar angle θ we have:

$$\frac{d\Gamma}{dx} = \frac{(M_H^2 - s)^3 sv}{2^{10}\pi^3 M_H}\left(1 + \frac{v^2}{3}\right)\{(1 - \lambda_1\lambda_2)[||A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2| +$$

$$+2Re(A_1A_3^* + A_2A_4^*)s_\gamma] + (\lambda_2 - \lambda_1)[2Re(A_1A_2^* + A_3A_4^*) + 2Re(A_1A_4^* + A_2A_3^*)s_\gamma]\}. \quad (28)$$

It follows that the non-zero $Re(A_1A_3^* + A_2A_4^*)$ and $Re(A_1A_2^* + A_3A_4^*)$ expressions give rise to the degree of circular polarization of the photon and the degree of longitudinal polarization of the fermion:

$$P_\gamma(s) = \frac{2Re(A_1A_3^* + A_2A_4^*)}{|A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2}, \quad (29)$$

$$P_f(s) = -\frac{2Re(A_1A_2^* + A_3A_4^*)}{|A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2}. \quad (30)$$

We obtained expressions for the degree of circular polarization of the photon $P_\gamma(s,z)$ and $P_\gamma(s)$, for the degree of longitudinal polarization of the fermion $P_f(s,z)$ and $P_f(s)$, and also for the forward-backward angular asymmetry $A_{FB}(s)$. Let us estimate them in the $H \Rightarrow \gamma + e^- + e^+$ decay, where the main Feynman diagrams are fermion and W-boson loop diagrams, and the bremsstrahlung diagrams are suppressed. The following parameters were used in the calculations: $M_H = 125$ GeV, $m_t = 173.2$ GeV, $m_e = 0.51 \cdot 10^{-3}$ GeV, $M_Z = 91.1875$ GeV, $\Gamma_Z = 2.4952$ GeV, $M_W = 80.385$GeV, $x_W = 0.2315$. It is assumed that particle loops are t-quark and the $W-$ boson.

Figure 7 shows the dependence of the degree of circular polarization of the photon on the invariant mass $\sqrt{s}$ at various angles θ .

As can be seen, at $\theta = 30°$ the degree of circular polarization of the photon is negative, with an increase in the energy of the fermion pair it decreases and reaches a minimum near $\sqrt{s} = 80$ GeV, and a further increase in energy leads to an increase in the degree of circular polarization of the photon. With increasing fermion emission angle, the degree of circular polarization of the photon module decreases. At $\theta = 90°$ the degree of circular polarization is zero.

Figure 8 illustrates the angular dependence of the degree of circular polarization of a photon at various $\sqrt{s}$.

It follows from the figure that at the fermion pair energy $\sqrt{s} = M_Z$, the degree of circular polarization of the photon is positive and decreases monotonically from 0.422 to 0.154 with increasing polar θ angle from zero to 180^0. However, at an energy $\sqrt{s} = 80$ GeV, the degree of circular polarization of the photon at the beginning of the angular spectrum is negative, increases monotonically with increasing angle θ and vanishes near 90°, and then the degree of circular polarization of the photon becomes positive and reaches a maximum at the end of the angular spectrum.

At an energy $\sqrt{s} = 40$ GeV, a similar dependence is also observed, however, the numerical value of $P_\gamma(s,z)$ is small and varies within the range of $-0.051 \le P_\gamma(s,z) \le 0.051$.

Figure 9 illustrates the angular dependence of the degree of longitudinal polarization of an electron $P_e(s,z)$ at various energies $\sqrt{s}$.

The graph shows that for $\sqrt{s} = M_Z$ at the beginning of the angular spectrum, the degree of longitudinal polarization of the electron is negative and monotonically increases from -0.422 to 0.154 with increasing angle θ.

At $\sqrt{s} = 80$GeV (40GeV), the degree of longitudinal polarization of the electron does not depend on the angle θ and contain 58.6% (5.1%).

As for the angular asymmetry of the forward and backward $A_{FB}(s)$, we note that in the $H \Rightarrow e^- + e^+ + \gamma$ decay this asymmetry due to $Re(A_1A_4^* + A_2A_3^*) \Rightarrow 0$ is equal to zero.

Figure 10 shows the dependence of the degree of circular polarization of the photon $P_\gamma(s)$ and the degree of longitudinal polarization of the electron $P_e(s)$ on the invariant mass $\sqrt{s}$. Due to $Re\ (A_1A_3^* + A_2A_4^*) = 0$, the degree of circular polarization is $P_\gamma(s) = 0$.

However, with an increase in the invariant mass of the e^-e^+ pair, the degree of longitudinal polarization of the electron increases and reaches a maximum near $\sqrt{s} = 80$ GeV, a further increase in the invariant mass leads to a decrease in the degree of longitudinal polarization of the electron.

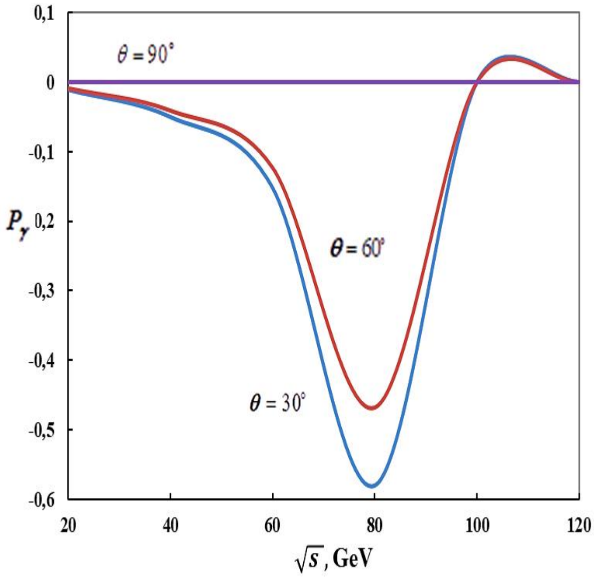

Fig. 7. The degree of circular polarization of photon in $H \Rightarrow e^- + e^+ + \gamma$ decay as a function of $\sqrt{s}$.

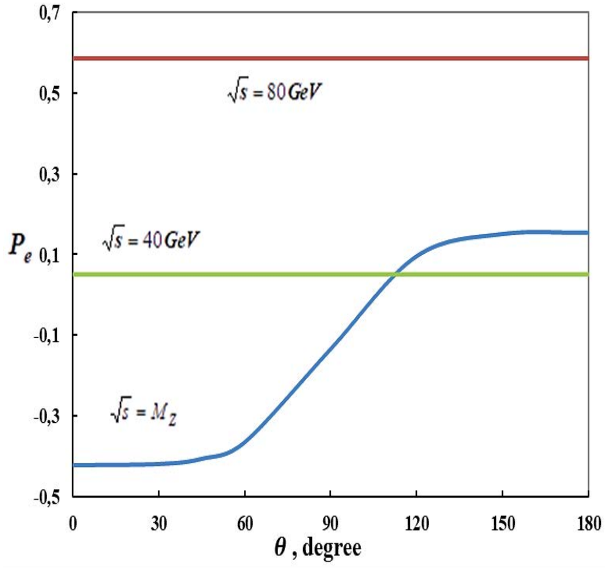

Fig. 9. Angular dependence of the degree of l ongitudinal polarization of electron $P_e(s,z)$ at various energies $\sqrt{s} = M_Z$.

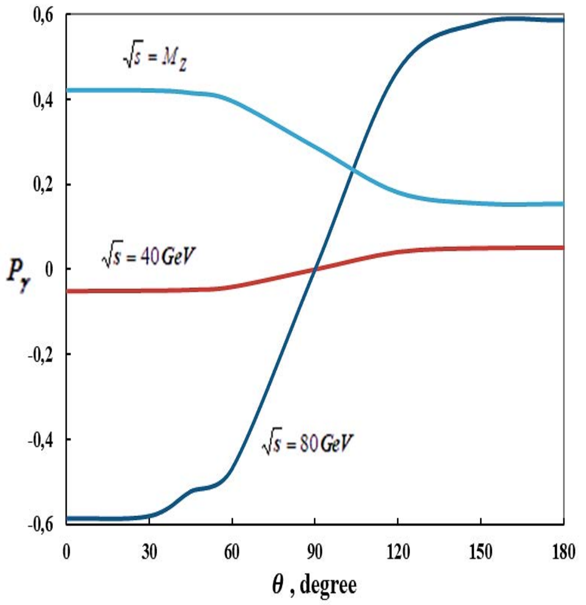

Fig. 8. Angular dependence $P_\gamma(s,z)$ in decay $H \Rightarrow e^- e^+ \gamma$.

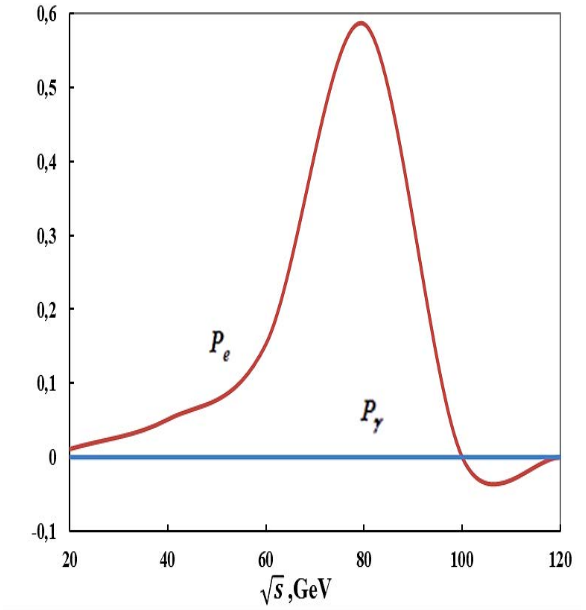

Fig. 10. The degrees of circular polarization of the photon $P_\gamma(s)$ and degree of longitudinal electron polarization $P_e(s)$ as a function of $\sqrt{s}$.

CONCLUSION

Within the framework of the SM, the radiation decay of standard Higgs boson $H \Rightarrow f + \bar{f} + \gamma$ was considered. The diagrams corresponding to the radiation of the photon by the fermion and antifermion, as well as the fermion and W - boson loop diagrams are studied in detail. Taking into account the longitudinal polarizations of the fermion pair and the circular polarization of the photon, an analytical expression is obtained for the decay width $H \Rightarrow f + \bar{f} + \gamma$. Expressions are found for the degree of circular polarization of the photon $P_\gamma(s,z)$, $P_\gamma(s)$, the degree of longitudinal polarization of the fermion $P_f(s,z)$, $P_f(s)$ and the angular asymmetry of the forward and backward $A_{FB}(s)$. Then, numerical estimates of these quantities in the $H \Rightarrow e^- + e^+ + \gamma$ decay are performed. The results are illustrated by graphs.

APPENDIX

Here we give the expression of the squared amplitude corresponding to the contribution of the fermion and W - boson loop diagrams:

$$|M_{LOOP}|^2 = |M_1|^2 + |M_2|^2 + M_1^* M_2 + M_2^* M_1, \quad (A.1)$$

$$|M_1|^2 = 2(p_1 \cdot k)(p_2 \cdot k)(e \cdot q)(e^* \cdot q)\{(1 - \lambda_1\lambda_2)[|A_1|^2 + |A_2|^2] + 2(\lambda_2 - \lambda_1) Re(A_1 A_2^*)\} + (k \cdot q)^2 \times$$
$$\times \{(1 - \lambda_1\lambda_2)[(|A_1|^2 + |A_2|^2)\big((p_2 \cdot e)(p_1 \cdot e^*) + (p_1 \cdot e)(p_2 \cdot e^*) - (p_1 \cdot p_2)(e^* \cdot e)\big) - 2Re(A_1 A_2^*) \times$$
$$\times\, i(p_1 p_2 e e^*)_\varepsilon] + (\lambda_2 - \lambda_1)[(|A_1|^2 + |A_2|^2) i(p_1 p_2 e^* e)_\varepsilon + 2Re(A_1 A_2^*)((p_2 \cdot e)(p_1 \cdot e^*) +$$
$$+(p_1 \cdot e)(p_2 \cdot e^*) - (p_1 \cdot p_2)(e^* \cdot e)]\} - (k \cdot q)(e \cdot q)\{(1 - \lambda_1\lambda_2)[(|A_1|^2 + |A_2|^2)((p_1 \cdot e^*)(p_2 \cdot k) +$$
$$+(p_2 \cdot e^*)(p_1 \cdot k)) - 2Re(A_1 A_2^*) i(p_1 p_2 k e^*)_\varepsilon] + (\lambda_2 - \lambda_1)[(|A_1|^2 + |A_2|^2) i(p_1 p_2 e^* k)_\varepsilon + 2Re(A_1 A_2^*) \times$$
$$\times (p_1 \cdot e^*)(p_2 \cdot k) + (p_2 \cdot e^*)(p_1 \cdot k))]\} - (k \cdot q)(e^* \cdot q)\{(1 - \lambda_1\lambda_2)[(|A_1|^2 + |A_2|^2)((p_2 \cdot e) \times$$
$$\times (p_1 \cdot k) + (p_1 \cdot e)(p_2 \cdot k)) + 2Re(A_1 A_2^*) i(p_1 p_2 k e)_\varepsilon] + (\lambda_2 - \lambda_1)[(|A_1|^2 + |A_2|^2) i(p_1 p_2 k e)_\varepsilon +$$
$$+2Re(A_1 A_2^*)((p_1 \cdot e)(p_2 \cdot k) + (p_2 \cdot e)(p_1 \cdot k))]\}; \quad (A.2)$$

$$|M_2|^2 = (1 - \lambda_1\lambda_2)\{[|A_3|^2 + |A_4|^2][2(p_1 p_2 k e^*)_\varepsilon (p_1 p_2 e k)_\varepsilon + (p_1 \cdot p_2)(\mu e^* k q)_\varepsilon (\mu e q k)_\varepsilon] +$$
$$+2Re(A_3 A_4^*)(\mu e^* k q)_\varepsilon (\nu e k q)_\varepsilon i(\mu \nu p_1 p_2)_\varepsilon\} + (\lambda_2 - \lambda_1)\{[|A_3|^2 + |A_4|^2] i(\mu e^* k q)_\varepsilon (\nu e k q)_\varepsilon (\mu \nu p_1 p_2)_\varepsilon +$$
$$+2Re(A_3 A_4^*)[2(p_1 p_2 k e^*)_\varepsilon (p_1 p_2 e k)_\varepsilon + (p_1 \cdot p_2)(\mu e^* k q)_\varepsilon (\mu e q k)_\varepsilon]\}; \quad (A.3)$$

$$M_1^* M_2 + M_2^* M_1 = (e \cdot q)\{(1 - \lambda_1\lambda_2)[(A_1^* A_3 + A_2^* A_4) i(p_1 p_2 k e^*)_\varepsilon((p_2 \cdot k) - (p_1 \cdot k)) +$$
$$+(A_1^* A_4 + A_2^* A_3)(\mu k q e^*)_\varepsilon (\mu p_1 p_2 k)_\varepsilon] + (\lambda_2 - \lambda_1)[(A_1^* A_3 + A_2^* A_4)(\mu k q e^*)_\varepsilon (\mu p_1 p_2 k)_\varepsilon +$$
$$+(A_1^* A_4 + A_2^* A_3) i(p_1 p_2 k e^*)_\varepsilon((p_2 \cdot k) - (p_1 \cdot k))]\} + (e^* \cdot q)\{(1 - \lambda_1\lambda_2)[(A_3^* A_1 + A_4^* A_2) \times$$
$$\times\, i(p_1 p_2 k e)_\varepsilon\big((p_1 \cdot k) - (p_2 \cdot k)\big) + (A_3^* A_2 + A_4^* A_1)(\nu e k q)_\varepsilon (\nu p_1 p_2 k)_\varepsilon] + (\lambda_2 - \lambda_1) \times$$
$$\times [(A_3^* A_1 + A_4^* A_2)(\nu e k q)_\varepsilon (\nu p_1 p_2 k e)_\varepsilon + (A_3^* A_2 + A_4^* A_1) i(p_1 p_2 k e)_\varepsilon\big((p_1 \cdot k) - (p_2 \cdot k)\big)]\} +$$
$$+(k \cdot q)\{(1 - \lambda_1\lambda_2)[(A_1^* A_3 + A_2^* A_4)((p_2 \cdot e) i(p_1 e^* k q)_\varepsilon + (p_1 \cdot e) i(p_2 e^* k q)_\varepsilon - (p_1 \cdot p_2) \times$$
$$\times\, i(e e^* k q)_\varepsilon) + (A_1^* A_4 + A_2^* A_3)(\mu e^* k q)_\varepsilon (p_2 e p_1 \mu)_\varepsilon - (A_3^* A_1 + A_4^* A_2)((p_2 \cdot e^*) i(p_1 e k q)_\varepsilon +$$
$$+(p_1 \cdot e^*) i(p_2 e k q)_\varepsilon - (p_1 \cdot p_2) i(e^* e k q)_\varepsilon) - (A_3^* A_2 + A_4^* A_1)(\nu e k q)_\varepsilon (\nu p_1 p_2 e^*)_\varepsilon] + (\lambda_2 - \lambda_1) \times$$
$$\times [(A_1^* A_3 + A_2^* A_4)(\mu e^* k q)_\varepsilon (\mu p_1 p_2 e)_\varepsilon + (A_1^* A_4 + A_2^* A_3)((p_2 \cdot e) i(p_1 e^* k q)_\varepsilon + (p_1 \cdot e) i(p_2 e^* k q)_\varepsilon -$$
$$-(p_1 \cdot p_2) i(e e^* k q)_\varepsilon) - (A_3^* A_1 + A_4^* A_2)(\nu e k q)_\varepsilon (\nu p_1 p_2 e^*)_\varepsilon - (A_3^* A_2 + A_4^* A_1)((p_2 \cdot e^*) i(p_1 e k q)_\varepsilon +$$
$$+(p_1 \cdot e^*) i(p_2 e k q)_\varepsilon - (p_1 \cdot p_2) i(e^* e k q)_\varepsilon)]\}. \quad (A.4)$$

Given designations are the following:

$$(abcd)_\varepsilon = \varepsilon_{\mu\nu\rho\sigma} a_\mu b_\nu c_\rho d_\sigma\ ,\ (\mu abc)_\varepsilon = \varepsilon_{\mu\nu\rho\sigma} a_\nu b_\rho c_\sigma\ ,\ (\mu\nu ab)_\varepsilon = \varepsilon_{\mu\nu\rho\sigma} a_\rho b_\sigma\ .$$

[1] *A. Djouadi*. The Anatomy of Electro-Weak Symmetry Breaking. Tome I. The Higgs Boson in the Standard Model. arXiv: hep–ph / 050317v.2, 2005.

[2] *S.K. Abdullaev.* Common properities of fundamental interactions. Baku, "Zəka print", 2018, p. 332 (in Azerbaijani).

[3] ATLAS Collaboration. Observation of a new particle in the search for the Standard Model Higgs boson at the ATLAS detector at the LHC. Phys. Letters, 2012. B 716, p. 1-29.

[4] CMS Collaboration. Observation of a new boson at mass of 125 GeV with the CMS experiment at the LHC. Phys. Letters, 2012, B 716, p. 30-61.

[5] *V.A. Rubakov.* UFN, 2012, t.182, №.10, p.1017-1025 (in Russian).

[6] *A.V. Lanev*. UFN, 2014, t. 184, № 9, p. 996-1004 (in Russian).

[7] *D.I. Kazakov.* UFN, 2014, t.184, № 9, p.1004-1017 (in Russian).

[8] *S.K. Abdullayev. M.Sh. Gojayev, F.A. Saddigh*. Decay channels of the Standard Higgs Boson. Moscow University Physics Bulletin, 2017, vol. 72, № 4, p. 329-339.

[9] *S.K. Abdullayev, E.Sh. Omarova*. Decays of Higgs Bosons into a gauge boson and a fermion-antifermion pair. Russian Physics Journal, 2019, vol. 62. № 1, p. 30-39.

[10] *S.K. Abdullayev, E.Sh. Omarova*. Decay channels of Higgs Bosons $H \Rightarrow \gamma\gamma$, $H \Rightarrow \gamma Z$, $H^{\pm} \Rightarrow \gamma W^{\pm}$. Journal of BEU, Physics, 2019, vol. 3, № 1, p. 39-57.

[11] *Yi Sun, H.-R. Chang, D.-N. Gao*. Higgs decays to $\gamma l^{+} l$ in the Standard Model. arXiv: 1303.2230v2, [hep-ph], 2013.

[12] *A. Abbasabadi, D. Browser-Chao, D.A. Dicus, W.W. Repko*. Radiative Higgs boson decays $H \Rightarrow f\bar{f}\gamma$. Phys. Rev. 1997, D55, p. 5647.

[13] *A. Abbasabadi, W.W. Repko*. Higgs boson decay to $\mu\bar{\mu}\gamma$. Phys. Rev. 2000, D62, p.054025.

[14] *R. Akbar, I. Ahmed, M.J. Aslam.* Lepton polarization asymmetries of $H \Rightarrow \gamma\tau^{+}\tau^{-}$ decay in the Standard Model. Prog. Theor. Exp. Phys., 20114, p. 093B03.

[15] *D.A. Dicus, W.W. Repko.* Calculation of the decay $H \Rightarrow e\bar{e}\gamma$. arXiv: 1302.2159 [hep-ph].

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