FOUR WAVE INTERACTION IN THE CONSTANT INTENSITY APPROXIMATION

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Parametrical interaction of waves with four frequencies in nondissipative negative index materials is studied by employment the constant intensity approximation, taking into account the reverse reaction of excited waves onto the exciting ones. An expression for complex amplitude as well as the efficiency of conversion to signal wave at the frequency ω_1 in arbitrary phase detuning is obtained. It is shown that in contrast to the constant field approximation both amplitude and conversion efficiency are the functions of intensities of forward pump waves as well as the weak wave at frequency ω_2 in the constant intensity approximation. It is shown that maxima or minima of the conversion efficiency displace with alteration of weak wave intensity. Analytical expression for the optimum value of phase mismatch at which reflective index reaches its maximum is derived.

Keywords : metamaterials, phase mismatch, for wave mixing, coefficient of reflection, constant intensity approximation.
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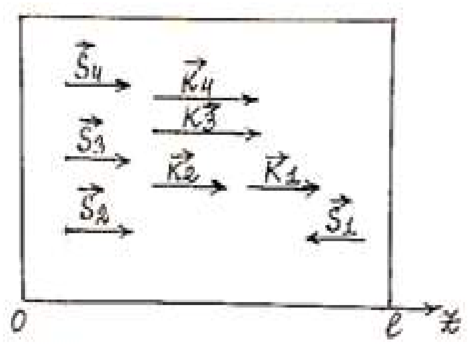
1. INTRODUCTION

The prospective negative index materials (NIM) are attractive with their unusual structure and properties [1,2]. The mesh of split-ring resonators are placed periodically. Each cavity is constructed of two concentric rings with a certain gap. The air gap between inner and outer ring serves as a capacitor. However the rings are act as an inductor making the LC resonant circuit.

Since NIM includes electro conducting wires-rods and split-ring resonator (Fig.1) the nonlinear response of material also has two components. Effective dielectric permittivity $\varepsilon = 1 - \omega_r^2/\omega$ where ω_r is the is the plasma frequency and ω is the frequency of the propagating electromagnetic wave. The effective permittivity is negative when the frequency is below the plasma frequency. When operating at the plasma frequency, the effective permittivity is zero, and hence yields a zero index of refraction. In a traditional nonlinear media four-frequency interaction are devoted number of works [3,4]. Investigations of nonlinear four-photon mixing in optical fibers are presented for the general case of depleted pump power in [5]. Efficiency of generation upon experimental four-frequency mixing in a layered negative index structure metal-dielectric-metal is higher as two orders as compared to that of generation in a pure gold film of 20nm thickness. Parametrical interactions of optical waves were investigated in metamaterials [6-8] by employment constant intensity approximation [9-13]. This paper centers on the study the four-frequency nonlinear mixing of optical waves in case of negligible linear losses .

2. THEORY AND DISCUSSIONS

We assume that idler A_2 and two pump waves $A_{3,4}$ are normally incident onto the left side surface of metamaterial with length l and propagate along positive direction of z- axis, while signal wave A_1 propagates in the opposite direction. Hence energy fluxes are as follows; $S_{2,3,4} -$ in thye positive direction of z-axis and S_1 in the negative direction (Fig.1).



When three light waves at frequencies ω_2 , ω_3 and ω_4 are incident onto cubic nonlinear medium the nonlinear polarization leads to the generation of a new electromagnetic with frequency $\omega_1 = \omega_3 + \omega_4 - \omega_2$. In a photon language the FWM is that two photons with initial frequencies are subjected to elastic scattering to produce two new photons. Here the law of conservation of energy and momentum have to be fulfilled in this process: $k_1 = k_3 + k_4 - k_2$ where $k_j -$ are the wave numbers at respective frequencies $\omega_j (j = 1 - 4)$. Conservation of momentum leads to the phase matching conditions. Boundary conditions of the described interaction of waves are given by $A_{2,3,4}(z = 0) = A_{20,30,40}$ and $A_1(z = l) = A_{1l}$, where $z = 0$ corresponds to the left input of the metamaterial, $A_{20,30,40}$ are the initial amplitudes of the transmitted weak wave (A_{20}) at the frequency ω_2 and of the pump waves ($A_{30,40}$) at the frequencies $\omega_{2,3}$ and A_{1l} is the initial amplitude of the transmitted signal wave at the right input of the nonlinear medium at $z =$

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l. For this consideration dielectric permittivity of the signal wave becomes negative that is reflected in the first equation of following set of reduced equations ($\delta_i = 0$):

$$\frac{dA_1}{dz} = -i\gamma_1 A_3 A_4 A_2^* e^{i\Delta z}, \quad \frac{dA_2}{dz} = i\gamma_2 A_3 A_4 A_1^* e^{i\Delta z}$$

$$\frac{dA_3}{dz} = i\gamma_3 A_1 A_2 A_4^* e^{-i\Delta z}, \quad \frac{dA_4}{dz} = i\gamma_4 A_1 A_2 A_3^* e^{-i\Delta z} \quad (1)$$

where A_j -are the complex amplitudes of the magnetic fields of the transmitted quasi-monochromatic waves, $\gamma_1 = 2\pi k_1 \chi_1^{(3)} / |\varepsilon_1|$ and $\gamma_{2,3,4} = 2\pi k_{2,3,4} \chi_{2,3,4}^{(3)} / \varepsilon_{2,3,4}$ are the nonlinear wave coupling coefficients, $\chi_j^{(3)}$ is the cubic susceptibility, and $\Delta = k_3 + k_4 - k_1 - k_2$ is the phase detuning of the interacting waves. The corresponding equations for the electric components can be derived analogously with replacement of the dielectric permittivity of the medium ε_j by the magnetic permeability μ_j and *vise versa* [14].

Differentiation of the first equation of system (1) yields to the following second order differential equatiuon

$$\frac{d^2A_1}{dz^2} - i\Delta\frac{dA_1}{dz} + (\gamma_1\gamma_2 I_{30} I_{40} - \gamma_1\gamma_3 I_{20} I_{40} - \gamma_1\gamma_4 I_{20} I_{30} + \Delta^2/4)A_1 = 0 \quad (2)$$

Having put $A_1 = a_1 \times \exp(i\Delta z/2)$ in the (2) gives

$$\frac{d^2a_1}{dz^2} + \lambda^2 a_1 = 0 \quad (3)$$

Where

$$\lambda = (\gamma_1\gamma_2 I_{30} I_{40} - \gamma_1\gamma_3 I_{20} I_{40} - -\gamma_1\gamma_4 I_{20} I_{30} + \Delta^2/4)^{1/2}$$

Solution of (3) is given by

$$a_1 = C_1 cos\lambda z + C_2 sin\lambda z \quad (4)$$

For a complex amplitude of signal wave we can write

$$A_1 = (C_1 cos\lambda z + C_2 sin\lambda z)\exp(i\Delta z/2) \quad (5)$$

Equation constants C_1 and C_2 are found by the boundary conditions:

$$A_1(z = l) = A_{1l} \text{ and } A_{2,3,4}(z = 0) = A_{20,30,40} \quad (6)$$

Employment of the first condition yields for constant C_1 :

$$C_1 = \frac{A_{1l} e^{-\frac{\Delta l}{2}}}{cos\lambda l} - C_2 tan\lambda l \quad (7)$$

Application the second condition to the first equation for C_2 gives

$$C_2 = -i[\gamma_1 A_{30} A_{40} A_2^* + (\Delta/2)C_1]/\lambda \quad (8)$$

Further substitutions has allowed to obtain complex amplitude of the signal wave in non-dissipative medium in the form

$$A_1(z) = (M_1 + iM_2)e^{\frac{\Delta z}{2}} \quad (9)$$

Here

$$M_1 = \frac{A_{1l} e^{-i\frac{\Delta l}{2}} cos\lambda z + m(\Delta/2)\frac{sin\lambda l}{\lambda}\frac{sin\lambda z}{\lambda}}{cos\lambda l - i\frac{\Delta}{2\lambda} sin\lambda l}$$

$$M_2 = \frac{m sin\lambda l \frac{cos\lambda z}{\lambda} - A_{1l} e^{-\frac{\Delta l}{2}}(\Delta/2)\frac{sin\lambda z}{\lambda}}{cos\lambda l - i\frac{\Delta}{2\lambda} sin\lambda l} - m\frac{sin\lambda z}{\lambda}$$

where $\lambda = (\gamma_1\gamma_2 I_{30} I_{40} - \gamma_1\gamma_3 I_{20} I_{40} - \gamma_1\gamma_4 I_{20} I_{30} + \Delta^2/4)^{1/2}$, $m = \gamma_1 A_{30} A_{40} A_2^*$

In the input $(z = 0)$ to the medium amplitude of signal wave is simplified as

$$A_1(z) = (M_3 + iM_4) \quad (10)$$

where

$$M_3 = A_{1l} e^{-i\frac{\Delta l}{2}} / (cos\lambda l - i\frac{\Delta}{2\lambda} sin\lambda l)$$

$$M_4 = m\frac{sin\lambda l}{\lambda} / (cos\lambda l - i\frac{\Delta}{2\lambda} sin\lambda l)$$

Having put Euler's formula $e^{-i\frac{\Delta l}{2}} = cos\frac{\Delta l}{2} - isin\frac{\Delta l}{2}$ into expression (10) we get amplification factor $\eta = I_1(z=0)/I_{1l}$ of a signal wave in case of arbitrary phase detuning:

$$\eta = \frac{(\frac{\Delta}{2\lambda}sin\lambda l \cdot sin\frac{\Delta l}{2} + cos\frac{\Delta l}{2}cos\lambda l - \frac{m\Delta sin^2\lambda l}{A_{1l}\ \ \lambda^2})^2 + (cos\frac{\Delta l}{2}\cdot\frac{\Delta}{2\lambda}sin\lambda l - sin\frac{\Delta l}{2}cos\lambda l + \frac{m}{A_{1l}}\frac{sin2\lambda l}{2\lambda})^2}{[\,cos^2\lambda l + (\frac{\Delta}{2\lambda})^2\, sin^2\lambda l]^2} \quad (11)$$

Energy exchange between interacting waves is mainly dependent on the phase matching conditions. When this condition satisfy enhancement of idler and signal waves occur because they extract energy from two pump waves. In opposite case energy can flow back from idler and signal waves to the pump waves. When two pumps and a signal wave are incident onto nonlinear medium the idler wave is generated through four-wave mixing. Phase matching is important for signal amplification and generation of idler wave.

In the left input ($A_{1l} = 0$) of nonlinear medium the coefficient of reflection of the mirror that is the metamaterial itself due to its negative refractive index is calculated from the (11) by the following expression:

$$R = \frac{I_1(z=0)}{I_{20}} = (\gamma_1 A_{30} A_{40} tan\lambda l)^2 \,/\, \left(\lambda^2 + \frac{\Delta^2}{4}\, tan^2\lambda l\right) \quad (12)$$

where

$$\lambda^2 = \frac{\Delta^2}{4} + K \;\;,\;\; K = \gamma_1\gamma_2 I_{30} I_{40} - \gamma_1\gamma_3 I_{20} I_{40} - \gamma_1\gamma_4 I_{20} I_{30}$$

To obtain the optimal value of phase mismatch parameter at which reflective index becomes maximum we differentiate expression (12) with respect to parameter Δ . Calculation shows that refractive index reaches its maxima when condition $\lambda l = 0,\ 1.5\pi,\ 2.5\pi,\ 3.5\,\pi$ is fulfilled. For optimal value of phase mismatch parameter then we get

$$\Delta_{opt,1} = 2\sqrt{\gamma_1\gamma_3 I_{20} I_{40} + \gamma_1\gamma_4 I_{20} I_{30} - \gamma_1\gamma_2 I_{30} I_{40}} \quad (13)$$

Principal maximum of reflective coefficient is obtained near to the $\Delta = 0$, which depends on the intensities of pump waves and idler wave.

Under phase matching conditions ($\Delta = 0$) we get from (10) more simplified expression for the amplitude of a signal wave :

$$A_1(z) = (M_5 + iM_6\,) \quad (14)$$

where $M_5 = A_{1l}/cos\lambda_1 l$, $M_6 = \frac{m}{\lambda} tan\lambda_1 l$, $\lambda_1 = (\gamma_1\gamma_2 I_{30} I_{40} - \gamma_1\gamma_3 I_{20} I_{40} - \gamma_1\gamma_4 I_{20} I_{30})^{1/2}$

All dependences of (10) and (12) on the various parameters of problem can be obtained in both constant field approximation and constant intensity approximation. Qualitative analysis of the formula (10) results in the following: conversion efficiency of energy of a signal wave has oscillator character which decrease with increase in the phase mismatch parameter. When coefficients γ_3 and γ_4 are differ from zero in the expression for λ we get result of constant intensity approximation while equality to zero is analogous to the result of constant field approximation.

As can be seen in contrast to the constant field approximation efficiency of conversion depends on the intensity of idler wave at frequency ω_2 in addition to other parameters. When we consider problem in the constant field approximation maxima and minima corresponding to different plots are coincide while those displace as a function of weak wave intensity in the constant intensity approximation.

Qualitative consideration shows that it is possible to obtain enhancement of the backward signal wave by varying the input intensities of forward waves.

3. CONCLUSIONS

On the basis of above stated one can conclude that efficiency of frequency conversion , coefficient of reflection upon four wave mixing in the constant intensity approximation are the functions of metamaterial thickness, intensities of forward pump waves as well as weak wave at frequency ω_2 . There is optimum value of phase mismatch at which coefficient of reflection reaches its maximum. In contrast to the constant field approximation , efficiency of frequency conversion increases with increase in the intensity of weak wave in the constant intensity approximation taking into account reverse reaction of excited waves on the phases of exciting waves. An existence a displacement of maxima and minima of conversion efficiency oscillations due to variations in intensity of weak wave has allowed to determine distance between neighbor minima or maxima and hence period of oscillations.

[1] *S.A. Akhmanov, V.A. Visloukh, A.S. Chirkin*. UFN 1986, v. 149, issue 3, p. 449-509.

[2] *Kh. Shumin, V. P. Drachev, A. V. Kildishev, et al.*, Nature, 2010, 466, 735–742.

[3] *K.K. Thyagarajan, Ajoy Ghatak*. Guided Wave Optical Components and devices. 2006.

[4] *F .Yaman, Govind P. Agrawal*. Guided Wave Optical Components and devices. 2006.

[5] *Yijiang Chen and Allan W.Snuder*. Optical letters, v.14, issue 1, 1989, pp. 87-89.

[6] *R.J. Kasumova, Z.H. Tagiyev, Sh.Sh. Amirov et al*., J. Russ. Laser Res., 2017, 38, 211–218.

[7] *R.J. Kasumova, Sh.Sh. Amirov, and A.A. Shamilova*. Kvant. Elektr., 2017, 47 (7), 655–660.

[8] *Z.A. Tagiev, R.J. Kasumova, and R.A. Salmanova*. Opt. Spektrosk., 1999, 87, 94–97.

[9] *Z.H. Tagiev, R.J. Kasumova, R.A. Salmanova, and N.V. Kerimova*. J. Opt. B, 2001, 3, 84–87.

[10] *R.J. Kasumova, Sh.Sh. Amirov*. Superlattices and microstructures Vol. 126, 2019, p. 49-56.

[11] *Sh.Sh. Amirov, Z.H. Tagiyev, G.N. Akhmedov*. AJP "Fizika" v. XXIV, 2018, N 4 p. 7-10.

[12] *Sh.Sh. Amirov, R.J. Kasumova, Z.H. Tagiev*. Proceedings of International conference "Modern Trends in Physics" 2019, p. 205-210.

[13] *Sh.Sh. Amirov, R.J .Kasumova, Z.H. Tagiev*. Proceedings of the VIII International scientific-practical conference "Science and practice: implementation to modern society ", Manchester, Great Britain December 26-28, 2020, Scientific Collection "Interconf". 2020, N 3 (39) p.1212-1214.

[14] *A.K. Popov and V.M. Shalaev*. Appl. Phys. B, 84, 2006, 131–137.

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