# SPECTRUM OF LASER PULSES IN THE FIRST ORDER DISPERSION THEORY

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Analytic expression for the spectral density of ultra short laser pulses in metamaterials (NIM) in the first order dispersion theory was obtained. Effects of group velocity delay (GVD) , the phase detuning as well as the phase modulation on the spectral density of signal wave are theoretically studied. Narrowing of the spectrum of signal wave and shift of its maxima toward smaller values of frequency with increase in the GVD are observed . At characteristic lengths differ from zero maxima of spectral density are obtained not at zero values of frequency but at its different values. It is shown, that an increase in phase detuning leads to decrease in the maxima of spectral density as well as to their shift toward larger values of frequency modulation parameter, but increase in group velocity delay leads to increase in the maxima and shift toward smaller values of frequency modulation parameter for the same phase detuning.

**Keywords:** Metamaterials, first order dispersion **,** group velocity delay, phase modulated pulse.
**PACS:**78.67.Pt ; 42.65-k; 42.62 Hk ; 42.70-a

## 1. INTRODUCTION

Amplification of weak signals and extension the radiation frequency range of laser requires the development of optical parametric amplifiers . An interest to development of powerful sources of light pulses of femtosecond duration is related to the analysis of nonstationary interaction of ultra short pulses. First theoretical investigations on the parametric amplifiers have been carried out by S.A. Akhmanov and R.V. Khokhlov [1]. The character of interaction of modulated waves significantly depends on the dispersion properties of medium. The frequency conversion for the ultra short pulses with running wave was considered in [2] .

The negative index materials (NIM) are attractive due to specifics of their interaction with electromagnetic waves. Different signs of refractive index correspond to different frequency intervals of the interacting waves. Therefore the energy fluxes of the waves with a positive sign of refractive index will propagate in opposite direction to those with frequencies corresponding to a negative sign of refractive index. The dynamics of three wave interaction in NIM was considered for the case of second harmonic generation in [3]. Results obtained in [4,5] are being used for the developments the metamaterials in the near IR and visible ranges of the spectrum. Earlier we have analyzed the efficiency of energy conversions between two direct waves with respect to the energy of the backward signal wave for the case of signal-wave amplification in metamaterials [6] in the constant intensity approximation (CIA) [7], taking into account the reverse reaction of excited wave on the exciting one. By employing the CIA we have studied the parametric interaction of optical waves in metamaterials under low-frequency pumping in the case of a negative index at a signal wave frequency [8]. The transition processes in the first order dispersion theory in the medium with quadratic nonlinearity have been analyzed by authors [9,10,11,12]. Earlier we have employed CIA to study the optical parametric amplification in stationary case [13] in the Fabri-Perrot cavity. Character of interaction of modulated wave significantly depends on the dispersion properties of a medium upon reduction of the pulse duration. The frequency conversion for the ultra-short pulses with running wave was analyzed in [14]. Earlier in [15] we were studied influence of group velocity delay (GVD) as well as group velocity dispersion (GVD) to the generation of sum frequency of ultra- short pulses in an external cavity under the phase matching and absence of linear losses. Latest times investigation of the four-wave interaction in metamaterials became actual. Analysis carried out in the CIA showed that in the metamaterial where the four-wave interaction occurs the optimum thickness of medium depends not only on the phase detuning and intensity of the pump wave but also on the intensity of a weak wave as well as the medium losses [16]. Efficiency of interaction between ultra short waves depends on the dispersion properties of the medium. Since metamaterials possess negative refractive index phase velocities are in the same direction distincly from the group velocities. In the dispersive medium difference between velocities of frequency components leads to the distortion of the pulse shape. This becomes more pronounced for the pulses with femtosecond duration . Analysis of the frequency transformation of laser pulses in metamaterials was carried out by us in the second order dispersion theory [17]. To study frequency transformation in metamaterials the CIA method was employed in [18-21]. Four wave parametrical interaction of waves in metamaterials was studied in [21].



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To study the three –wave parametric interaction in metamaterials in the first order dispersion theory is the goal of this paper.

## 2. THEORY AND DISCUSSIONS

It is assumed that waves interact in nonlinear medium when two pump and weak waves advance in the positive direction of z-axis ( $z = 0$ ) and the signal wave propagates in the opposite direction of the axis from ($z = l$) input surface. During this interaction the energy exchange between occurs between pump ($A_2(t,z)$ ), weak ($A_3(t,z)$ ) and signal ($A_1(t,z)$ ) waves and this wave is parametrically amplified. Dielectric permittivity and magnetic permeability of the medium at the frequency of signal wave are negative but those are positive at frequencies $\omega_2$ and $\omega_3$ for the pump and weak waves respectively.

Three-wave interaction of waves in the second order dispersion theory is described by the following set of truncated equations [1].

$$\left(\frac{\partial(z,t)}{\partial z}+\frac{1}{u_1}\frac{\partial}{\partial t}-i\frac{g_1}{2}\frac{\partial^2}{\partial t^2}+\delta_1\right)A_1 = i\gamma_1 A_3 A_2^* e^{i\Delta z}$$

$$\left(\frac{\partial}{\partial z}+\frac{1}{u_2}\frac{\partial}{\partial t}-i\frac{g_2}{2}\frac{\partial^2}{\partial t^2}+\delta_2\right)A_2 = -i\gamma_2 A_3 A_1^* e^{i\Delta z} \tag{1}$$

$$\left(\frac{\partial}{\partial z}+\frac{1}{u_3}\frac{\partial}{\partial t}-i\frac{g_3}{2}\frac{\partial^2}{\partial t^2}+\delta_1\right)A_3 = -i\gamma_3 A_1 A_2 e^{-i\Delta z}$$

Here $A_j$ (j=1-3)- are the complex amplitudes of a signal, pump and idler waves respectively , $\delta_j -$ are the absorption coefficients of the medium at frequencies $\omega_j$ (j=1-3), $u_j -$ are the group velocities of the interacting waves , $\Delta = k_1 - k_2 - k_3$ is the phase mismatch between the interacting waves , $g_j = \partial^2 k_j / \partial\omega_j^2$ is the dispersion of group velocities and $\gamma_1$ , $\gamma_2$ , $\gamma_3$, are the coefficients of nonlinear coupling. Since we study the problem in the first order dispersion theory the second order derivatives will disappear in above equations. In addition if consider the amplitude of pump wave to be fixed ( $A_2 = A_{20} = const.$) then we can rewrite:

$$\left(\frac{\partial(z,t)}{\partial z}+\frac{1}{u_1}\frac{\partial(z,t)}{\partial t}+\delta_1\right)A_1 = i\gamma_1 A_3 A_2^* e^{i\Delta z}$$

$$\left(\frac{\partial(z,t)}{\partial z}+\frac{1}{u_3}\frac{\partial(z,t)}{\partial t}+\delta_3\right)A_3 = -i\gamma_3 A_1 A_2 e^{-i\Delta z} \tag{2}$$

If we introduce new variables $\eta = t - \frac{z}{u_1}$, $\nu = 1/u_3 - 1/u_1$ (is a group velocity delay) we get following system with respect to $z$ and $\eta$.

$$\left(\frac{\partial(z,\eta)}{\partial z}-\delta_1\right)A_1(z,\eta) = i\gamma_1 A_3(z,\eta) A_2^* e^{i\Delta z}$$

$$\left(\frac{\partial(z,\eta)}{\partial z}-\nu\frac{\partial(z,\eta)}{\partial \eta}+\delta_3\right)A_3(z,\eta) = -i\gamma_3 A_1(z,\eta) A_2 e^{-i\Delta z} \tag{3}$$

Employment the Fourier transformation $A_{1,2}(z,\eta) = \int_{-\infty}^{+\infty} A_{1,2}(z,\omega)e^{-i\omega\eta}d\omega$ to the system yields

$$\left(\frac{\partial(z,\omega)}{\partial z}-\delta_1\right)A_1(z,\omega) = i\gamma_1 A_3(z,\omega) A_2^* e^{i\Delta z}$$

$$\left(\frac{\partial(z,\omega)}{\partial z}+i\omega\nu+\delta_3\right)A_3(z,\omega) = -i\gamma_3 A_1(z,\omega) A_2 e^{-i\Delta z} \tag{4}$$

Solving the system (4) according to the problem boundary conditions $A_1(z = l) = 0$ and $A_{2,3}(z = 0) = A_{20}A_{30}$ yields for the complex amplitude of signal wave $A_1(z,\omega)$ ($\delta_i = 0$)

$$A_1(\omega,z) = \frac{i\gamma_1 A_{20} A_3}{\lambda - h\tan\lambda_1 l}(\sin\lambda z - \tan\lambda l \cdot \cos\lambda z)e^{-hz} \tag{5}$$

where $\lambda^2 = \Gamma_3^2 + \frac{1}{4}(\omega\nu - \Delta)^2$ , $h = i(\omega\nu - \Delta)$ , $\Gamma_3 = {l_{nl}}^{-1}$

We consider the idler wave to be Gaussian with second order of phase modulation in the temporary domain.

$$A_{30}(t) = A_0 \exp[-\left(\frac{1}{2\tau^2} + i\frac{\gamma}{2}\right)t^2] \tag{6}$$

By employment the inverse Fourier transformation we obtain spectral density of idler wave in the frequency domain

$$A_3(\omega) = \frac{A_{20}\tau^2}{2\pi}\frac{1}{\sqrt{1+p}}e^{-\frac{\mu^2}{1+p}} \tag{7}$$

where $\mu = \omega\tau$ and $p = \gamma^2\tau^4$ are the frequency and phase modulation parameters respectively. Substitution expression (7) into (5) for the spectral density ($S_1(\omega,z) = A_1(\omega,z)\cdot A_1^*(\omega,z)$) of signal wave yields

$$S_1(\omega,z) = K\frac{e^{-\frac{\mu^2}{1+p}}(sin\lambda z - tan\lambda l\cdot cos\lambda z)^2}{(\lambda z)^2 + (hz)^2 tan^2\lambda l} \tag{8}$$

where $K = \frac{cn\gamma_1^2 I_{30} I_{20}\tau^2 z^2}{16\pi}$, where $\lambda = l_{nl}{}^{-1}[\frac{1}{4}\left(\frac{l_{nl.}}{l_\nu}\omega\nu - \frac{\Delta}{\Gamma_3}\right)^2 - 1]^{1/2}$, $h = l_{nl}{}^{-1}[i(\frac{l_{nl.}}{l_\nu}\omega\nu - \frac{\Delta}{\Gamma_3})]$

From (8) it follows that the shape of a spectrum of a amplified signal wave is determined not only by the values of $z$, $l_{nl}$, and $l_\nu$ but also with their ratios as the $z/l_{nl}$, and $l_{nl}/l_\nu$. In Fig. 1 the dependences of a reduced spectral density $\widetilde{S_1}(\omega,z)$ on the parameter $\omega\tau$ are illustrated at different values of $l_{nl}/l_\nu$.

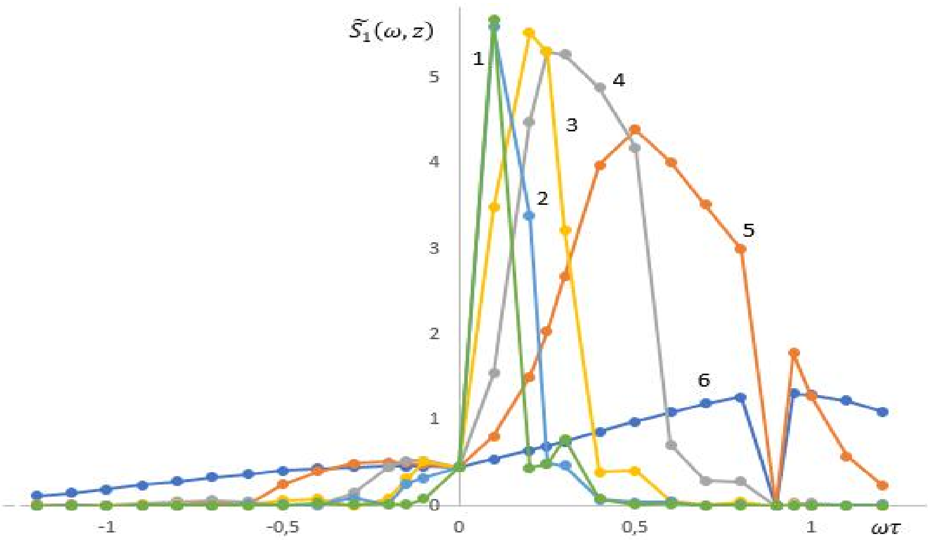


*Fig. 1.* Dependence of reduced spectral density $\widetilde{S_1}(\omega,z) = S_1(\omega,z)/K$ of a signal wave on the parameter $\omega\tau$ for $p = 0$, $z/l_{nl} = 0{,}5$, $\frac{\Delta l_{nl}}{2} = 1$, $\delta_i = 0$ and various values of $l_{nl}/l_\nu$ : $1 - l_{nl}/l_\nu = 20$ ; $2 - l_{nl}/l_\nu = 15$ ; $3 - l_{nl}/l_\nu = 10$ ; $4 - l_{nl}/l_\nu = 6$ ; $5 - l_{nl}/l_\nu = 3$ ; $6 - l_{nl}/l_\nu = 1$

As can be seen, the shape of a spectrum varies with variation in this ratio . Comparison of plots demonstrates that spectrum of signal wave becomes narrowed and its maxima shift toward smaller values of parameter $\omega\tau$ with increase in the group velocity mismatch . However this behavior is not observed for larger values of $l_{nl}/l_\nu$ ( Fig.1.plots 1 and 2 )

Fig.2 illustrates plots of reduced spectral density as a function of parameter $\omega\tau$ for various values of ratio $z/l_{nl}$.

As can be seen maxima of spectral density decrease with increase in $z/l_{nl}$. An increase in phase modulation also leads to decrease in the maximum of spectral density ( plots 1 and 3 ).

Dependences of reduced spectral density versus parameter $\omega\tau$ for different values of phase detuning ($\frac{\Delta l_{nl}}{2}$) are presented in Fig.3 . It is seen that when there is no phase detuning plot of spectrum becomes symmetric with respect to frequency modulation parameter $\omega\tau$. An increase in phase detuning leads to decrease in the maxima as well as to shift the maxima toward larger values of $\omega\tau$. Also can be seen that increase $l_{nl}/l_\nu$ leads to increase in the maxima and shift toward smaller values of $\omega\tau$ for the same phase detuning (plots 2 and 3 ). Positive value of phase detuning parameter promote higher value of spectral density with respect to the same negative value (plots 2 and 5).

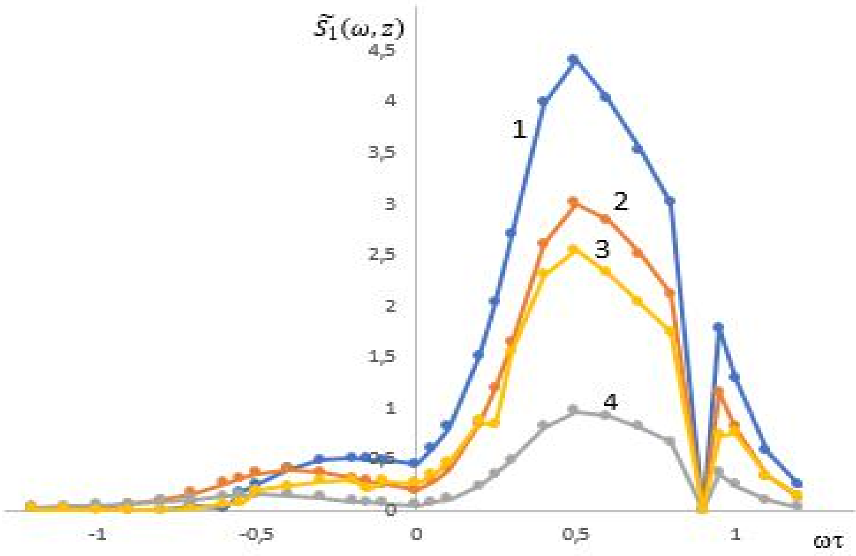


*Fig. 2.* The reduced spectral density $\widetilde{S_1}(\omega,z) = S_1(\omega,z)/K$ of a signal wave as a function of parameter $\omega\tau$ for $p = 0$ (curves 1, 2 and 4 ), $p = 3$ (curve 3) , $l_{nl}/l_v$ =3 , $\frac{\Delta l_{nl}}{2} = 1$, $\delta_i = 0$ and various values of $z/l_{nl}$ : $1 - z/l_{nl} = 0{,}5$; $2 - z/l_{nl} = 1$; $3 - z/l_{nl} = 0{,}5$; $4 - z/l_{nl} = 1{,}5$

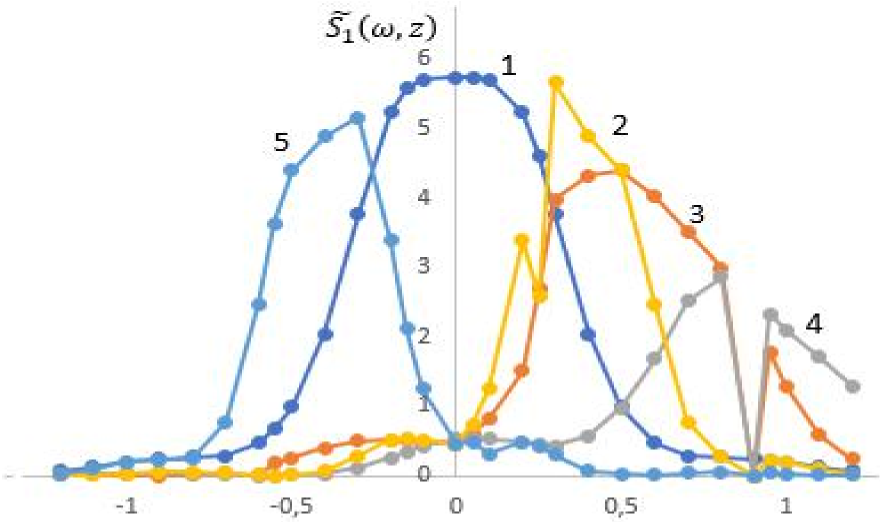


*Fig. 3.* Dependence of the reduced spectral density $\widetilde{S_1}(\omega,z) = \frac{S_1(\omega,z)}{K}$ of a signal wave versus parameter $\omega\tau$ for $p = 0$ , $\frac{l_{nl}}{l_v}$ =3 (plots 1, 3, 4) , $\frac{l_{nl}}{l_v}$ =5 (plots 2 and 5) $\delta_i = 0$ , $z / l_{nl} = 0{,}5$ and various values of parameter $\frac{\Delta l_{nl}}{2}$ : $1 - \frac{\Delta l_{nl}}{2} = 0$ ; $2 - \frac{\Delta l_{nl}}{2} = 1$; $3 - \frac{\Delta l_{nl}}{2} = 1$; $4 - \frac{\Delta l_{nl}}{2} = 1{,}5$; $5 - \frac{\Delta l_{nl}}{2} = -1$

## 3. CONCLUSIONS

From above mentioned one can conclude that parametric amplification of ultra-short pulses in metamaterial in the first order dispersion theory is affected by the group velocity mismatch as well as the phase detuning between interacting waves. We showed that the spectral density of ultra-short pulse wave is affected by the ratios of characteristic lengths. When there is phase matching, the shape of a graph of the spectral density becomes symmetric relatively negative and positive values of frequency modulation parameter. For the ratios of characteristic lengths $l_{nl}/l_v$ differ from zero maxima of spectral density are obtained not at zero values of frequency but at its different values. Spectrum of signal wave becomes narrowed and its maxima shift toward smaller values of frequency with increase in the group velocity mismatch. This behavior promotes development of generators with narrow frequency band. An increase in phase modulation leads to decrease in the maximum of spectral density. An increase in phase detuning leads to decrease in the maxima as well as to shift the maxima toward larger values of frequency modulation. An increase in group velocity mismatch leads to increase in the maxima and shift toward smaller values of frequency modulation for the same phase detuning. Positive value of phase detuning parameter increases spectral density with respect to the same negative value .


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*Received: 01.03.2021*