INVESTIGATION OF CP-ODD ASYMMETRIES IN MUON COLLIDERS

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In the framework of the Minimal Supersymmetric Standard Model, the CP-odd asymmetries in the process of fermionic pair generation in muon colliders are investigated: $\mu^-\mu^+ \to f\bar{f}$. Taking into account the arbitrary polarizations of the muon-antimuon and fermion-antifermion pairs, a general expression for the effective cross-section of the process is obtained. Expressions are found for CP-odd asymmetries associated with the longitudinal and transverse polarizations of the muon-antimuon pair, as well as the degrees of longitudinal (transverse) polarization of the fermion and antifermion. Studying these characteristics can provide valuable information about the nature of the Higgs boson.

Keywords: muon-antimuon pair, Minimal Supersymmetric Standard Model, Higgs boson, degree of longitudinal (transverse) polarization, longitudinal (transverse) spin asymmetry, top quark.
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1. INTRODUCTION

The Standard Model (SM) of strong and electroweak interactions has achieved great success in describing various processes of high-energy physics. Especially with the discovery of the Higgs boson H_{SM} in the Large Hadron Collider (LHC) by the ATLAS and CMS collaborations [1, 2] (see also reviews [3-5]), a new era in high-energy physics began. On the one hand, the mechanism of generating the masses of fundamental particles – the mechanism of spontaneous violation of the Braut-Engler-Higgs symmetry has been experimentally confirmed [6-8]. On the other hand, the theory of fundamental interactions has received a logical conclusion and has acquired the status of a standard theory. Within the framework of the SM, Feynman diagrams of various processes can be considered and compared with the corresponding experimental data. The agreement between the SM and the experiment is strikingly good.

Despite the success of SM, this theory has its own difficulties. One of the difficulties is related to the renormalization of the Higgs boson mass. The fact is that for all SM particles, mass renormalization works well, and in the case of the Higgs boson, a problem arises: the vacuum has a strong influence on the mass of the Higgs boson, its mass increases by trillions of times, and such a particle can no longer play the role of the Higgs boson. There is no restraining factor inside the SM that stops the growth of the Higgs boson mass due to virtual particles. Such a way out of a difficult situation is possible here. If there are some other particles in nature that are absent in the SM, then they can compensate for the influence of the Higgs boson on the mass in a virtual form. The most important thing here is that in models of physics outside the SM, for example, in the Minimal Supersymmetric Standard Model (MSSM), such compensation occurs by itself after the construction of the theory.

The absence of dark matter particles in the SM is also one of the difficulties of this theory. Astrophysicists believe that in the Universe, in addition to ordinary matter in the form of planets, stars, black holes,
gas and dust clouds, neutrinos, etc., there are particles of a completely different nature, which we do not see in any range of electromagnetic waves. These are dark matter particles that practically do not interact with ordinary matter and radiation. There is not a single particle in the SM that is suitable for this role. In the MSSM, there are such particles as neutralinos, sneutrinos, gluinos, gravitinos, which can be candidates for dark matter.

The universe consists almost entirely of matter, individual planets, stars, galaxies consisting of antimatter are not observed. Such an imbalance of matter over antimatter should have arisen dynamically at the earliest stages of the evolution of the Universe. However, calculations show that the SM is unable to generate the necessary imbalance. In fact, the very existence of the world as we see it speaks of the insufficiency of SM. The above facts, as well as other reasons, indicate going beyond the scope of the SM. At the same time, special attention is paid to the MSSM [9-11].

In this model, two scalar field doublets with hypercharges -1 and +1 are introduced:

$$\varphi_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \ \varphi_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}.$$

In order to obtain the physical fields of the Higgs bosons, the fields φ_1 and φ_2 are represented as

$$\varphi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \upsilon_1 + H_1^0 + iP_1^0 \\ H_1^- \end{pmatrix}, \ \varphi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} H_2^+ \\ \upsilon_2 + H_2^0 + iP_2^0 \end{pmatrix}$$

where H_1^0, P_1^0, H_2^0 and P_2^0 are the fields describing the excitations of the system with respect to vacuum states $<\varphi_1>= \frac{1}{\sqrt{2}}\upsilon_1$ and $<\varphi_2>= \frac{1}{\sqrt{2}}\varphi_2$.

CP even Higgs bosons H and h are obtained by mixing the fields H_1^0 and H_2^0 (mixing angle α):

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}.$$

Similarly, the fields P_1^0 and P_2^0, $H_1^\pm$ and $H_2^\pm$ are mixed (mixing angle β):

$$\begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} P_1^0 \\ P_2^0 \end{pmatrix},$$

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$$\begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} H_1^{\pm} \\ H_2^{\pm} \end{pmatrix}.$$

Here G^0 and $G^{\pm}$ are neutral and charged Goldstone bosons, $A(H^{\pm})$ – CP are odd (charged) Higgs bozones.

Consequently, five Higgs bosons appear in the MSSM: CP-even H and h bosons, CP-odd A-boson and charged $H^{\pm}$-bosons.

The Higgs sector is characterized by mass parameters M_h, M_H, M_A, $M_{H^{\pm}}$, and angular parameters α and β. Of these, the parameters are considered free, M_A and $\mathrm{tg}\beta = \frac{\upsilon_2}{\upsilon_1}$ the remaining parameters are expressed through them:

$$M_{H(h)}^2 = \frac{1}{2}[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\,],$$

$$M_{H^{\pm}}^2 = M_A^2 + M_W^2, \ \mathrm{tg}2\alpha = \mathrm{tg}2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}\left(-\frac{\pi}{2} \le \alpha \le 0\right),$$

where M_Z and M_W are the masses of the gauge bosons Z and $W^{\pm}$.

Muon colliders are widely discussed in order to study the physical properties of both the standard Higgs boson H_{SM} and the Higgs bosons, H, h and A the MSSM [12-15]. In these colliders, Higgs bosons can be born directly in the s-channel during the annihilation of muon-antimuon pairs $\mu^-\mu^+ \to (H^*, h^*, A^*) \to f\bar{f}$, where $f\bar{f}$ -fermion-antifermion pair (lepton pair $\tau^-\tau^+$, quark pair $c\bar{c}$, $b\bar{b}$, $t\bar{t}$).

Note that collisions of polarized leptons and antileptons (electrons and positrons e^-e^+, muons and antimuons $\mu^-\mu^+$) at high energies are an effective method for studying the structure of elementary particles and the mechanisms of their interaction. This is mainly due to the following circumstances. *First*, the interaction of muons and antimuons (electrons and positrons) is described by the electroweak Weinberg-Salam theory, so the results obtained are well interpreted. *Secondly*, since muons and antimuons (electrons and positrons) do not participate in strong interactions, the background conditions of experiments are significantly improved compared to studies conducted with proton beams in the LHC.

It should be noted that high-energy electron-positron and muon-antimuon colliders are either designed or planned to be designed in various laboratories around the world. These include the electron-positron International Linear Collider (ILC) with an energy of 0.5 TeV in the center of mass system [13, 16], the Compact Linear Collider (CLIC) with an energy of 3 TeV [14], the Muon Collider (MC) with an energy of 1.5-4 TeV [13]. These colliders will help solve a number of issues related to the physics of the Higgs boson.

In this paper, the CP-odd asymmetries in the process of the production of a pair of fundamental fermions during the annihilation of a muon-antimuon pair are investigated within the framework of the MSSM

$$\mu^- + \mu^+ \to f + \bar{f} \qquad (1)$$

Here $f\bar{f}$ there can be a pair of tau-leptons $\tau^-\tau^+$, a pair of quarks $c\bar{c}$ and $b\bar{b}$. The production of a heavier top quark pair will be investigated separately. The success of the accelerator technology on counter $\mu^-\mu^+$ (e^-e^+)-beams makes it already real to study this process in a wide range of energies. Taking into account the arbitrary polarization states of both the initial and final particles, an analytical expression for the effective cross-section of the process is obtained, the CP-odd asymmetries due to the longitudinal and transverse polarizations of the muon-antimuon pair, the degrees of longitudinal (transverse) polarization of the fermion-antifermion pair are determined.

2. THE AMPLITUDE AND CROSS-SECTION OF THE PROCESS $\mu^-\mu^+ \to (\Phi^*) \to f\bar{f}$

In the framework of the MSSM, the process (1) is described by the Feynman diagrams a) and b) shown in fig. 1. In the lowest order of perturbation theory, annihilation $\mu^-\mu^+ \to f\bar{f}$ can occur both through a virtual Higgs boson Φ^* ($\Phi^* = H^*$, h^* or A^*), and through a virtual photon and Z^*-boson.

The amplitude corresponding to diagram a) can be written in the following form (we believe that the boson has CP-even and CP-odd components)

$$M(\mu^-\mu^+ \to (\Phi^*) \to f\bar{f}) = \frac{m_\mu m_f}{\upsilon^2} D_\Phi(s) \cdot g_{\Phi\mu\mu} g_{\Phi ff} [\bar{\upsilon}(p_2, s_2)(a_\mu + \gamma_5 b_\mu) u(p_1, s_1)] \times$$
$$\times [\bar{u}(k_1, r_1)(a_f + \gamma_5 b_f) \upsilon(k_2, r_2)], \qquad (2)$$

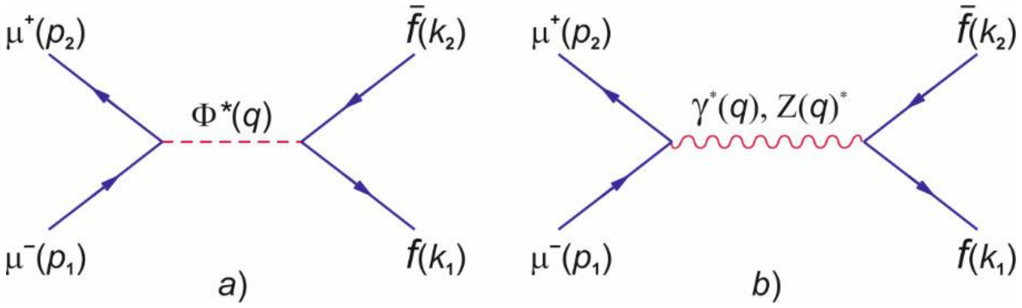

Fig. 1. Feynman diagrams of the reaction $\mu^-\mu^+ \to f\bar{f}$

where $D_\Phi(s) = (s - M_\Phi^2 + iM_\Phi\Gamma_\Phi)^{-1}$ – is the propagator of the Φ-boson; M_Φ and Γ_Φ – is the mass and total width of this boson; $p_1(s_1)$, $p_2(s_2)$, $k_1(r_1)$ and $k_2(r_2)$ – 4 are the impulses (polarization vectors) of the muon, antimuon, fermion and antifermion, respectively; $s = (p_1 + p_2)^2 = q^2$ – the square of the total energy of the muon-antimuon pair in the center of mass system; m_μ and m_f – the masses of the muon and fermion; $\upsilon = 246$ GeV$=(\sqrt{2}G_F)^{-1/2}$ – the vacuum value of the standard Higgs boson field; a_μ, b_μ, a_f and b_f – some complex constants; $g_{\Phi\mu\mu}$ and $g_{\Phi ff}$ – the interaction constants of the Higgs boson Φ with the muon-antimuon and fermion-antifermion pair, normalized to the interaction constants of the standard Higgs boson (they are shown in Table 1).

Table 1.

Interaction constants $g_{\Phi ff}$

Φ - бозон	$g_{\Phi\mu\mu}$	$g_{\Phi cc}$	$g_{\Phi bb}$	$g_{\Phi tt}$
H	$\frac{\cos\alpha}{\cos\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\cos\beta}$	$\frac{\sin\alpha}{\sin\beta}$
h	$-\frac{\sin\alpha}{\cos\beta}$	$\frac{\cos\alpha}{\sin\beta}$	$-\frac{\sin\alpha}{\cos\beta}$	$\frac{\cos\alpha}{\sin\beta}$
A	tgβ	ctgβ	tgβ	ctgβ

Squaring the matrix element of the reaction $\mu^-\mu^+ \to (\Phi^*) \to f\bar{f}$, we get the expression (the masses of the muon and fermion are neglected in comparison with their energies):

$$\left|M(\mu^-\mu^+ \to (\Phi^*) \to f\bar{f})\right|^2 = \left(\frac{m_\mu m_f}{\upsilon^2}\right)^2 g_{\Phi\mu\mu}^2 g_{\Phi ff}^2 \left|D_\Phi(s)\right|^2 \times$$
$$\times\{[\left|a_\mu\right|^2 + \left|b_\mu\right|^2][(p_1\cdot p_2) + m_\mu^2(s_1\cdot s_2)] + [\left|a_\mu\right|^2 - \left|b_\mu\right|^2][(p_1\cdot s_2)(p_2\cdot s_1) - (p_1\cdot p_2)(s_1\cdot s_2)] +$$
$$+ 2\,\mathrm{Re}(a_\mu b_\mu^*) m_\mu[(p_1\cdot s_2) + (p_2\cdot s_1)] + 2\,\mathrm{Im}(a_\mu b_\mu^*)\varepsilon_{\mu\nu\rho\sigma} p_{1\mu} p_{2\nu} s_{2\rho} s_{1\sigma}\}\times$$
$$\{[\left|a_f\right|^2 + \left|b_f\right|^2][(k_1\cdot k_2) + m_f^2(r_1\cdot r_2)] + [\left|a_f\right|^2 - \left|b_f\right|^2][(k_1\cdot r_2)(k_2\cdot r_1) - (k_1\cdot k_2)(r_1\cdot r_2)] +$$
$$+ 2\,\mathrm{Re}(a_f b_f^*) m_f[(k_1\cdot r_2) + (k_2\cdot r_1)] - 2\,\mathrm{Im}(a_f b_f^*)\varepsilon_{\mu\nu\rho\sigma} k_{1\mu} k_{2\nu} r_{2\rho} r_{1\sigma}\} \qquad (3)$$

The differential cross-section of the process $\mu^-\mu^+ \to (\Phi^*) \to f\bar{f}$ in the case of arbitrary polarizations of the muon-antimuon and the fermion-anti-fermion pair in the center-of-mass system can be represented as

$$\frac{d\sigma}{d\Omega}(\mu^-\mu^+ \to (\Phi^*) \to f\bar{f}) = \frac{N_C}{256\pi^2}\cdot\left(\frac{m_\mu m_f}{\upsilon^2}\right)^2 g_{\Phi\mu\mu}^2 g_{\Phi ff}^2 \left|D_\Phi(s)\right|^2 s\times$$
$$\{\left|a_\mu\right|^2[1 + (\vec{\xi}_1\vec{\xi}_2) - 2(\vec{n}\vec{\xi}_1)(n\vec{\xi}_2)] + \left|b_\mu\right|^2[1 - (\vec{\xi}_1\vec{\xi}_2)] - 2\,\mathrm{Re}(a_\mu b_\mu^*)[(\vec{n}\vec{\xi}_1) - (n\vec{\xi}_2)] - 2\,\mathrm{Im}(a_\mu b_\mu^*)(\vec{n}[\vec{\xi}_1\vec{\xi}_2])\}\times$$
$$\times\{\left|a_f\right|^2[1 + (\vec{\eta}_1\vec{\eta}_2) - 2(\vec{n}_0\vec{\eta}_1)(n_0\vec{\eta}_2)] + \left|b_f\right|^2[1 - (\vec{\eta}_1\vec{\eta}_2)] + 2\,\mathrm{Re}(a_f b_f^*)[(\vec{n}_0\vec{\eta}_1) - (\vec{n}_0\vec{\eta}_2) - 2\,\mathrm{Im}(a_f b_f^*)(\vec{n}_0[\vec{\eta}_1\vec{\eta}_2])\}. \qquad (4)$$

Here N_C – is the colour factor (in the case of the production of a tau-lepton pair $N_C = 1$, and at production $c\bar{c}$ or $b\bar{b}$ a quark pair $N_C = 3$) $\vec{\xi}_1$ and $\vec{\xi}_2$ ($\vec{\eta}_1$ and $\vec{\eta}_2$) are the unit vectors directed along the spin vectors of the muon and antimuon (fermion and antifermion) in their rest systems, $\vec{n}$, $\vec{\eta}_0$ – are the unit vectors along the muon and fermion impulses. We analyze the differential effective cross-section (4) in various cases of polarizations of the muon-antimuon and fermion-antifermion pairs.

3. THE CASE OF LONGITUDINAL POLARIZATION OF PARTICLES

Suppose that the muon-antimuon and the fermion-antifermion pair are longitudinally polarized

$$(\vec{n}\vec{\xi}_1)=\lambda_1\,,\ (\vec{n}\vec{\xi}_2)=-\lambda_2\,,\ (\vec{\xi}_1\vec{\xi}_2)=-\lambda_1\lambda_2\,,\ (\vec{\eta}_0\vec{\eta}_1)=h_1\,,\ (\vec{n}_0\vec{\eta}_2)=-h_2\,,\ (\vec{\eta}_1\vec{\eta}_2)=-h_1h_2\,,$$

where λ_1 and λ_2 (h_1 and h_2) are the helicities of the muon and antimuon (fermion and antifermion). In this case, the total effective cross-section of the process $\mu^-\mu^+\to(\Phi^*)\to f\bar{f}$ will take the form:

$$\sigma(\mu^-\mu^+\to(\Phi^*)\to f\bar{f})=\frac{N_C}{64\pi}\left(\frac{m_\mu m_f}{\upsilon^2}\right)^2|D_\Phi(s)|^2 s(g_{\Phi\mu\mu}g_{\Phi ff})^2\times$$
$$\times[(|a_\mu|^2+|b_\mu|^2)(1+\lambda_1\lambda_2)-2\,\mathrm{Re}(a_\mu b_\mu^*)(\lambda_1+\lambda_2)][(|a_\mu|^2+|b_\mu|^2)(1+h_1h_2)+2\,\mathrm{Re}(a_f b_f^*)(h_1+h_2)] \quad (5)$$

It follows from this formula that the muon and the antimuon (fermion and antifermion) must have the same helicities:

$$\lambda_1=\lambda_2=\pm1\,,\ h_1=h_2=\pm1$$

This is a consequence of preserving the full moment in the annihilation process $\mu^-\mu^+\to(\Phi^*)\to f\bar{f}$. Indeed, let us consider this process in the center-of-mass system. In this system, the muon and antimuon (fermion and antifermion) impulses are equal in magnitude and opposite in direction (see Figure 2, where the directions of the impulses and spins of the muon-antimuon and fermion-antifermion pair are shown). Since the Higgs boson Φ spin is zero, the process $\mu^-\mu^+\to(\Phi^*)\to f\bar{f}$ is allowed only due to the fact that the muon and antimuon, as well as the fermion and antifermion are in a state with the same helicities. It is in this case that the projection of the total moment of the muon and antimuon (fermion and antifermion) on the direction of the muon momentum is zero.

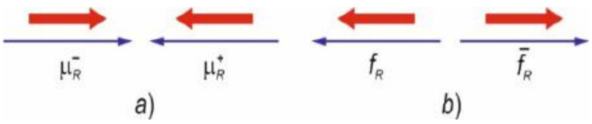

Fig. 2. Directions of impulses and spins of particles in the process $\mu^-\mu^+\to(\Phi^*)\to f\bar{f}$

According to formula (5), diagram (a) of Fig. 1 corresponds to four spiral sections:

1) all particles are right polarized:

$$\sigma_{RR}=\frac{N_C}{16\pi}\left(\frac{m_\mu m_f}{\upsilon^2}\right)^2|D_\Phi(s)|^2 s(g_{\Phi\mu\mu}g_{\Phi ff})^2|a_\mu-b_\mu|^2|a_f+b_f|^2\,;$$

2) all particles are left polarized:

$$\sigma_{LL}=\frac{N_C}{16\pi}\left(\frac{m_\mu m_f}{\upsilon^2}\right)^2|D_\Phi(s)|^2(g_{\Phi\mu\mu}g_{\Phi ff})^2|a_\mu+b_\mu|^2|a_f-b_f|^2\,;$$

3) the $\mu^-\mu^+$-pair is polarized right, a $f\bar{f}$ – to the left:

$$\sigma_{RL}=\frac{N_C}{16\pi}\left(\frac{m_\mu m_f}{\upsilon^2}\right)^2|D_\Phi(s)|^2(g_{\Phi\mu\mu}g_{\Phi ff})^2|a_\mu-b_\mu|^2|a_f-b_f|^2\,;$$

4) the $\mu^-\mu^+$-pair is polarized to the left, and $f\bar{f}$ – to the right:

$$\sigma_{LR}=\frac{N_C}{16\pi}\left(\frac{m_\mu m_f}{\upsilon^2}\right)^2|D_\Phi(s)|^2(g_{\Phi\mu\mu}g_{\Phi ff})^2|a_\mu+b_\mu|^2|a_f+b_f|^2\,.$$

Here, the first (second) index of the spiral section corresponds to the helicities of the muon and antimuon (fermion and antifermion). In these cases, the directions of the pulses and spins of the muon-antimuon and fermion-antifermion pairs are shown in Fig. 3. As can be seen from the figure, the directions of the spins of the muon-antimuon (fermion-antifermion) pair are oriented oppositely, therefore, the sum of their spins is zero, the spin of the intermediate Φ-boson is also zero.

Let us perform summation in the effective section (5) over the polarization states of the fermion and antifermion and present the resulting expression in the following form

$$\sigma(\lambda_1,\lambda_2)=\frac{N_C}{16\pi}\left(\frac{m_\mu m_f}{\upsilon^2}\right)^2|D_\Phi(s)|^2 s(g_{\Phi\mu\mu}g_{\Phi ff})^2[|a_f|^2+|b_f|^2]\times$$
$$\times[(|a_\mu|^2+|b_\mu|^2)(1+\lambda_1\lambda_2)-2\,\mathrm{Re}(a_\mu b_\mu^*)(\lambda_1+\lambda_2)] \quad (6)$$

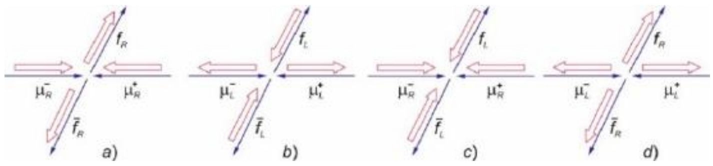

Fig. 3. Directions of impulses and spins of particles in the reaction $\mu^-\mu^+ \to (\Phi^*) \to f\bar{f}$

We define the longitudinal spin asymmetries due to the polarization of the muon and antimuon:

$$A_{\lambda_1} = \frac{1}{\lambda_1}\cdot\frac{\sigma(\lambda_1,0)-\sigma(-\lambda_1,0)}{\sigma(\lambda_1,0)+\sigma(-\lambda_1,0)} = -\frac{2\,\mathrm{Re}(a_\mu b_\mu^*)}{|a_\mu|^2+|b_\mu|^2}, \quad (7)$$

$$A_{\lambda_2} = \frac{1}{\lambda_2}\cdot\frac{\sigma(0,\lambda_2)-\sigma(0,-\lambda_2)}{\sigma(\lambda_2,0)+\sigma(0,-\lambda_2)} = -\frac{2\,\mathrm{Re}(a_\mu b_\mu^*)}{|a_\mu|^2+|b_\mu|^2}. \quad (8)$$

Here $\sigma(\lambda_1,0)$ ($\sigma(0,\lambda_2)$) – is the effective cross-section of the process under consideration, when the muon (antimuon) is longitudinally polarized, and the antimuon (muon) is not polarized. It follows from expressions (7) and (8) that the longitudinal spin asymmetry A_{λ_1}, that occurs during the annihilation of a polarized muon and an unpolarized antimuon is equal to the longitudinal spin asymmetry A_{λ_2}, that occurs during the annihilation of an unpolarized muon and a polarized antimuon.

An experimental study of these longitudinal spin asymmetries can provide valuable information about the CP-odd nature of the Φ-boson. If the Φ-boson is a CP-even particle, like H- or h-boson ($a_\mu = 1$, $b_\mu = 0$) or a CP-odd particle, like A-boson ($a_\mu = 0$, $b_\mu = 1$), then no longitudinal spin asymmetry will be detected in experiments: $A_{\lambda_1} = A_{\lambda_2} = 0$.

Information about the CP-odd Φ-boson can also be obtained by studying the degree of longitudinal polarization of the fermion or antifermion. Indeed, in (5) we perform summation over the polarizations of the antifermion and averaging over the polarization states of the muon-antimuon pair, we present the resulting expression as

$$\sigma(h_1) = \frac{1}{2}\sigma_0(1+h_1 P_f), \quad (9)$$

where σ_0 – is the total effective cross-section of the process $\mu^-\mu^+ \to (\Phi^*) \to f\bar{f}$ in the case of unpolarized particles

$$\sigma_0 = \frac{N_C}{16\pi}\left(\frac{m_\mu m_f}{\upsilon^2}\right)^2 |D_\Phi(s)|^2 s(g_{\Phi\mu\mu}g_{\Phi ff})^2[|a_\mu|^2+|b_\mu|^2][|a_f|^2+|b_f|^2], \quad (10)$$

a P_f – is the degree of longitudinal polarization of the fermion

$$P_f = \frac{2\,\mathrm{Re}(a_f b_f^*)}{|a_f|^2+|b_f|^2} \quad (11)$$

It is convenient to study this degree of longitudinal polarization during the production of a tau-lepton pair during the annihilation of a muon-antimuon pair. This is due to the possibility of measuring the tau-lepton polarization by studying decays $\tau^- = \pi^-\nu_\tau$, $\tau^- = K^-\nu_\tau$, $\tau^- = \rho^-\nu_\tau$. The widths of these decays are very sensitive to the polarization of the lepton, which allows us to measure this polarization in experiments.

If we assume that the real and imaginary parts of the constants a_μ, b_μ, a_τ and b_τ are equal to each other ($Re\, a_\mu = Im\, a_\mu = Re\, b_\mu = Im\, b_\mu = Re\, a_\tau = Im\, a_\tau = Re\, b_\tau = Im\, b_\tau$), then the longitudinal spin asymmetry A_{λ_1} and the degree of longitudinal polarization of the tau-lepton P_τ can reach values of the order ± 1.

4. THE CASE OF TRANSVERSE POLARIZATION OF PARTICLES

As is known, electrons and positrons moving in storage rings acquire mainly transverse polarization due to synchrotron radiation [17]. In the case when the initial and final particles are transversely polarized, the differential cross-section of the process $\mu^-\mu^+ \to (\Phi^*) \to f\bar{f}$ has the form:

$$\frac{d\sigma}{d\Omega}(\mu^-\mu^+ \to \Phi^* \to f\bar{f}) = \frac{N_C}{256\pi^2}\left(\frac{m_\mu m_f}{\upsilon^2}\right)^2 (g_{\Phi\mu\mu}g_{\Phi ff})^2 |D_\Phi(s)|^2 s\{|a_\mu|^2+|b_\mu|^2+(|a_\mu|^2-|b_\mu|^2)\xi_1^\perp\xi_2^\perp\cos\phi -$$
$$-2\,\mathrm{Im}(a_\mu b_\mu^*)\xi_1^\perp\xi_2^\perp\sin\phi\}[|a_f|^2+|b_f|^2+(|a_f|^2-|b_f|^2)\eta_1^\perp\eta_2^\perp\cos\varphi - 2\,\mathrm{Im}(a_f b_f^*)\eta_1^\perp\eta_2^\perp\sin\varphi] \quad (12)$$

Here $\xi_1^\perp$ and $\xi_2^\perp$ ($\eta_1^\perp$ and $\eta_2^\perp$) are the transverse components of the spin vectors of the muon and antimuon (fermion and antifermion) (with full transverse polarization $\xi_1^\perp = \xi_2^\perp = \eta_1^\perp = \eta_2^\perp = 1$), ϕ (φ) – is the angle between the vectors $\vec{\xi}_1^\perp$ and $\vec{\xi}_2^\perp$ ($\eta_1^\perp$ and $\eta_2^\perp$).

The differential effective cross-section (12) leads to the following transverse spin symmetries associated with the polarizations of the muon-antimuon pair (with full transverse polarization $\xi_1^1 = \xi_2^1 = 1$)

$$A_1 = \frac{d\sigma(\phi = 0)/d\Omega - d\sigma(\phi = \pi)/d\Omega}{d\sigma(\phi = 0)/d\Omega + d\sigma(\phi = \pi)/d\Omega} = \frac{|a_\mu|^2 - |b_\mu|^2}{|a_\mu|^2 + |b_\mu|^2}, \quad (13)$$

$$A_2 = \frac{d\sigma(\phi = -\pi/2)/d\Omega - d\sigma(\phi = \pi/2)/d\Omega}{d\sigma(\phi = -\pi/2)/d\Omega + d\sigma(\phi = \pi/2)/d\Omega} = \frac{2\,\mathrm{Im}(a_\mu b_\mu^*)}{|a_\mu|^2 + |b_\mu|^2} \quad (14)$$

From the formula of the effective cross-section (12), we can also determine the degrees of transverse polarization of the fermion-antifermion pair according to the definition:

$$P_1 = \frac{d\sigma(\varphi = 0)/d\Omega - d\sigma(\varphi = \pi)/d\Omega}{d\sigma(\varphi = 0)/d\Omega + d\sigma(\varphi = \pi)/d\Omega} = \frac{|a_f|^2 - |b_f|^2}{|a_f|^2 + |b_f|^2}, \quad (15)$$

$$P_2 = \frac{d\sigma(\varphi = -\pi/2)/d\Omega - d\sigma(\varphi = \pi/2)/d\Omega}{d\sigma(\varphi = -\pi/2)/d\Omega + d\sigma(\varphi = \pi/2)/d\Omega} = \frac{2\,\mathrm{Im}(a_f b_f^*)}{|a_f|^2 + |b_f|^2}. \quad (16)$$

For a complete transversely polarized muon-antimuon (fermionantifermion) pair, we have $A_1 = 1$ ($P_1 = 1$) if the Φ-boson is a CP-even Higgs boson H or h: $a_\mu = 1$, $b_\mu = 0$ ($a_f = 1$, $b_f = 0$). For a CP-odd A-boson $a_\mu = 0$, $b_\mu = 1$ ($a_f = 0$, $b_f = 1$) the transverse spin asymmetry (the degree of transverse polarization) is $A_1 = -1$ ($P_1 = -1$). Therefore, by measuring the transverse spin asymmetry A_1 or the degree of transverse polarization P_1, information about the nature of the Higgs boson Φ can be obtained. The difference from zero of the transverse spin asymmetry A_2 or the degree of transverse polarization P_2 also indicates a violation of CP-parity in the reaction $\mu^-\mu^+ \to (\Phi^*) \to f\bar{f}$.

We determine the total effective annihilation cross section of a muon-antimuon pair into a fermion-antifermion pair, taking into account the contribution of all diagrams with the exchange of CP-even Higgs bosons H and h, as well as CP-odd A-boson (for H- and h-bosons $a_\mu = a_f = 1$, $b_\mu = b_f = 0$, and for A-boson $a_\mu = a_f = 0$, $b_\mu = b_f = 1$): $\mu^-\mu^+ \to (H^*, h^*, A^*) \to f\bar{f}$. At the same time, due to different CP-parity, there is no interference between H- and A-bosons, as well as between h- and A-bosons. As a result, for the effective cross-section of the reaction $\mu^-\mu^+ \to (H^*, h^*, A^*) \to f\bar{f}$, the expression was obtained

$$\sigma_0(\mu^-\mu^+ \to (H^*, h^*, A^*) \to f\bar{f}) = \frac{N_C}{16\pi} \cdot \left(\frac{m_\mu m_f}{\upsilon^2}\right)^2 s\{|D_H(s) g_{H\mu\mu} g_{Hff} + D_h(s) g_{h\mu\mu} g_{hff}|^2 + |D_A(s)|^2 (g_{A\mu\mu} g_{Aff})^2\}. \quad (17)$$

Fig. 4 shows the energy dependence of the effective cross-section of the process $\mu^-\mu^+ \to (H^*, h^*, A^*) \to \tau\bar{\tau}$ at $M_A = 400$ GeV, $\mathrm{tg}\beta = 3$, $\Gamma_h = \Gamma_H = \Gamma_A = 4$ GeV, $m_\mu = 0.1056$ GeV, $m_\tau = 1.778$ GeV, $M_Z = 91.1875$ GeV. As can be seen, with an increase in energy $\sqrt{s}$, the effective cross-section increases and reaches a maximum at $\sqrt{s} = M_A$ =400 GeV, and a further increase in energy leads to a decline in the effective cross-section

5. THE AMPLITUDE AND CROSS-SECTION OF THE REACTION $\mu^-\mu^+ \to (\Phi^*) \to t\bar{t}$

We investigated the process of the production of a $\tau^-\tau^+$-lepton pair or $c\bar{c}$ ($b\bar{b}$)-quark pair during the annihilation of a muon-antimuon pair. At the same time, we neglected the masses of the muon and fermion compared to their energies ($m_\mu^2 \ll s$, $m_f^2 \ll s$). However, when a top quark pair is born during the annihilation of a muon-antimuon pair, we cannot neglect the mass of the t-quark, since this mass is too large ($m_t = 173.2$ GeV) [18]:

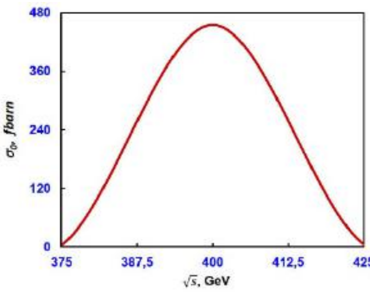

Fig. 4. Dependence of the effective cross-section of the reaction $\mu^-\mu^+ \to \tau\bar{\tau}$ on the energy

$$M(\mu^-\mu^+\to(\Phi^*)\to t\bar{t})=\frac{m_\mu m_t}{\upsilon^2}D_\Phi(s)g_{\Phi\mu\mu}g_{\Phi tt}[\bar{\upsilon}(p_2,s_2)(a_\mu+\gamma_5 b_\mu)u(p_1,s_1)]\times$$
$$\times[\bar{u}(k_1,r_1)(a_t+\gamma_5 b_t)\upsilon(k_2,r_2)] \quad (18)$$

Based on this amplitude, the following expression is obtained for the effective cross-section of the process $\mu^-\mu^+\to(\Phi^*)\to t\bar{t}$, when the muon-antimuon pair and the top quark pair are longitudinally polarized:

$$\sigma(\mu^-\mu^+\to(\Phi^*)\to t\bar{t})=\frac{|D_\Phi(s)|^2}{64\pi}\left(\frac{m_\mu m_t}{\upsilon^2}\right)^2(g_{\Phi\mu\mu}g_{\Phi tt})^2 s\sqrt{1-\frac{4m_t^2}{s}}\{[|a_\mu|^2+|b_\mu|^2](1+\lambda_1\lambda_2)-$$
$$-2\,\mathrm{Re}(a_\mu b_\mu^*)(\lambda_1+\lambda_2)\}\left\{[|a_t|^2\left(1-\frac{4m_t^2}{s}\right)+|b_t|^2](1+h_1h_2)+2\,\mathrm{Re}(a_t b_t^*)\sqrt{1-\frac{4m_t^2}{s}}(h_1+h_2)\right\} \quad (19)$$

From this effective cross-section, it can be seen that the muon and antimuon, as well as the top quark and top antiquark, as well as at the production of a $\tau^-\tau^+$ -lepton pair or, $c\bar{c}$, ($b\bar{b}$)-quark pair, must have the same helicities $\lambda_1=\lambda_2=\pm1$, $h_1=h_2=\pm1$. The longitudinal spin asymmetry caused by the polarization of the muon (antimuon) $A_{\lambda_1}(A_{\lambda_2})$ coincides with the formula (7), (8) obtained in the reaction $\mu^-\mu^+\to(\Phi^*)\to\tau^-\tau^+$. However, the degree of longitudinal polarization of the top quark in the form

$$P_t=\frac{2\,\mathrm{Re}(a_t b_t^*)\sqrt{1-4m_t^2/s}}{|a_t|^2(1-4m_t^2/s)+|b_t|^2} \quad (20)$$

it differs significantly from the degree of longitudinal polarization of the tau-lepton in the reaction $\mu^-\mu^+\to(\Phi^*)\to\tau^-\tau^+$. The degree of longitudinal polarization of the top quark depends not only on the parameters a_t and b_t, but also on the energy of the counter muon-antimuon beams.

Figure 5 shows the dependence of the degree of longitudinal polarization of the top quark on the energy $\sqrt{s}$ at $\mathrm{Re}\,a_t=\mathrm{Im}\,a_t=\mathrm{Re}\,b_t=\mathrm{Im}\,b_t$ and $m_t=173.2$ GeV. As can be seen, the degree of longitudinal polarization of the top quark is positive and it increases with an increase in the energy of the counter muon-antimuon beams.

Note that the longitudinal polarization of the top quark has already been measured in the LHC by the ATLAS detector at the production of a quark pair in proton-proton collisions with an energy of 7 TeV in the center of mass system [19].

Now let's consider the case when the muon-antimuon pair and the top-quark pair are transversely polarized. In this case, the differential cross-section of the process $\mu^-\mu^+\to(\Phi^*)\to t\bar{t}$ is determined by the expression:

$$\frac{d\sigma}{d\Omega}(\mu^-\mu^+\to(\Phi^*)\to t\bar{t})=\left(\frac{m_\mu m_t}{\upsilon^2}\right)^2\frac{N_C}{256\pi^2}(g_{\Phi\mu\mu}g_{\Phi tt})^2|D_\Phi(s)|^2 s\sqrt{1-\frac{4m_t^2}{s}}\times$$
$$\times\left\{[|a_\mu|^2+|b_\mu|^2]+(|a_\mu|^2-|b_\mu|^2)\xi_1^\perp\xi_2^\perp\cos\phi-2\,\mathrm{Im}(a_\mu b_\mu^*)\xi_1^\perp\xi_2^\perp\sin\phi\}\{[|a_t|^2+|b_t|^2]\left[1-\frac{2m_t^2}{s}(1+\eta_1^\perp\eta_2^\perp\cos\varphi)\right]+\right.$$
$$\left.+[[|a_t|^2-|b_t|^2]\left[\eta_1^\perp\eta_2^\perp\cos\varphi-\frac{2m_t^2}{s}[1+\eta_1^\perp\eta_2^\perp\cos\varphi]-2\,\mathrm{Im}(a_t b_t^*)\sqrt{1-\frac{4m_t^2}{s}}\eta_1^\perp\eta_2^\perp\sin\varphi\right]\right\}. \quad (21)$$

From this effective cross-section, we determine the degree of transverse polarization of the top quark pair:

$$P_t^\perp=\frac{d\sigma(\varphi=0)/d\Omega-d\sigma(\varphi=\pi)/d\Omega}{d\sigma(\varphi=0)/d\Omega+d\sigma(\varphi=\pi)/d\Omega}=\frac{|a_t|^2(1-4m_t^2/s)-|b_t|^2}{|a_t|^2(1-4m_t^2/s)+|b_t|^2}. \quad (22)$$

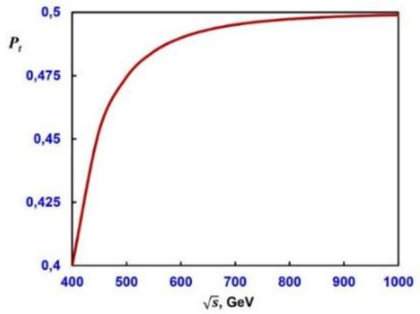

Fig. 5. Energy dependence of the degree of longitudinal polarization of the top quark in the reaction $\mu^-\mu^+\to(\Phi^*)\to t\bar{t}$

Fig. 6 shows the energy dependence of the degree of transverse polarization of the top quark at $\mathrm{Re}\, a_t = \mathrm{Im}\, a_t = \mathrm{Re}\, b_t = \mathrm{Im}\, b_t$, $m_t = 173.2$ GeV. It follows from the figure that the degree of transverse polarization of the top quark is negative and it increases with an increase in the energy of the counter muon-antimuon beams.

Now we consider the integral cross-sections of reactions $\mu^-\mu^+ \to (H^*) \to t\bar{t}$ and $\mu^-\mu^+ \to (A^*) \to t\bar{t}$ in the case of unpolarized particles. In the process $\mu^-\mu^+ \to (H^*) \to t\bar{t}$, due to the CP-parity of the Higgs boson $a_\mu = a_t = 1$, $b_\mu = b_t = 0$, are taken, and in the process $\mu^-\mu^+ \to (A^*) \to t\bar{t}$, due to the CP-odd of the A-boson $a_\mu = a_t = 0$, $b_\mu = b_t = 1$:

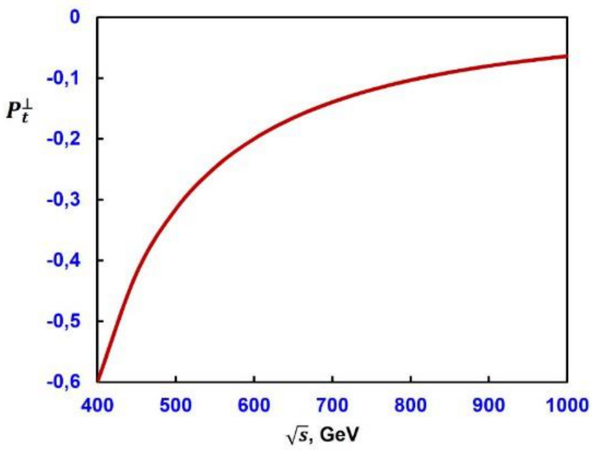

Fig. 6. Dependence of the degree of transverse polarization of a $t\bar{t}$ -quark pair in the process of $\mu^-\mu^+ \to t\bar{t}$.

$$\sigma(\mu^-\mu^+ \to (H^*) \to t\bar{t}) = \left(\frac{m_\mu m_t}{\upsilon^2}\right)^2 \frac{s}{(s - M_H^2)^2 + M_H^2\Gamma_H^2} \frac{1}{16\pi} \left(\frac{\sin 2\alpha}{\sin 2\beta}\right)^2 \left(1 - \frac{4m_t^2}{s}\right)^{3/2},$$

$$\sigma(\mu^-\mu^+ \to (A^*) \to t\bar{t}) = \left(\frac{m_\mu m_t}{\upsilon^2}\right)^2 \frac{s}{(s - M_A^2)^2 + M_A^2\Gamma_A^2} \frac{1}{16\pi} \sqrt{1 - \frac{4m_t^2}{s}}.$$

Fig. 7 shows the dependence of the cross sections of the processes $\mu^-\mu^+ \to (H^*) \to t\bar{t}$, $\mu^-\mu^+ \to (A^*) \to t\bar{t}$ on the energy $\sqrt{s}$ at $M_A = 500$ GeV, $\mathrm{tg}\beta = 3$. It can be seen that with an increase in energy $\sqrt{s}$, the cross-section of the process $\mu^-\mu^+ \to (H^*) \to t\bar{t}$ increases and reaches a maximum at $\sqrt{s} = M_H = 500$ GeV, and a further increase in energy $\sqrt{s}$ leads to a decrease in the cross-section. A similar dependence is observed in the process $\mu^-\mu^+ \to (A^*) \to t\bar{t}$, but the maximum of the cross-section is shifted towards high energies.

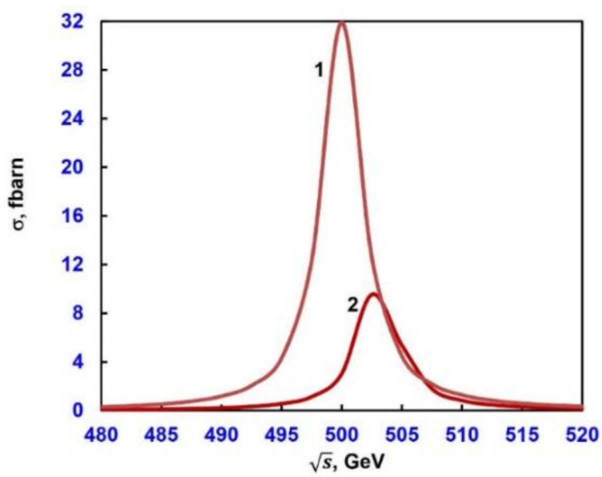

Fig. 7. Energy dependence of the cross sections of reactions $\mu^-\mu^+ \to (H^*) \to t\bar{t}$ (1) and $\mu^-\mu^+ \to (A^*) \to t\bar{t}$ (2)

6. THE AMPLITUDE AND CROSS-SECTION OF THE REACTION $\mu^-\mu^+ \to (\gamma^*; Z^*) \to t\bar{t}$

As noted above, the process of annihilation of a muon-antimuon pair into a pair of fundamental fermions can go both through a virtual photon and through a virtual Z -boson (see diagram b) in Figure 1). The diagram with photon exchange is responsible for the production of a fermion-antifermion pair with negative C-parity, since the photon is charge-odd. The diagram with the exchange Z -boson corresponds to the production of a pair $f\bar{f}b$ of superposition states with $C = +1$ and $C = -1$, since the vertex Z -boson contains both vector and axial interaction. The interference of states with charge parity ± 1 should give an asymmetry in the angular distribution of the fermion and antifermion. First, we will discuss the qualitative properties of processes with longitudinally polarized initial and final particles. It is known that the helicity of fermions is preserved in vector and axial interactions [20]. The preservation of helicity requires that the muon and the antimuon have opposite helicities ($\mu_L^-\mu_R^+$ or $\mu_R^-\mu_L^+$). The same is true for a fermion-antifermion pair in the final state: $f_L \bar{f}_R$ or $f_R \bar{f}_L$. Therefore, at high energies, only four independent spiral amplitudes should correspond to the process $\mu^-\mu^+ \to f\bar{f}$: F_{LL}, F_{LR} F_{RL} and F_{RR} (the first and second indices of these spiral amplitudes correspond to the spiralities of the muon μ^- and fermion f) which describe the following spiral processes [21]:

$$\mu_L^- + \mu_R^+ \to f_L + \bar{f}_R, \quad \mu_L^- + \mu_R^+ \to f_R + \bar{f}_L,$$

$$\mu_R^- + \mu_L^+ \to f_L + \bar{f}_R, \quad \mu_R^- + \mu_L^+ \to f_R + \bar{f}_L .$$

We define these spiral amplitudes by writing the amplitudes $F_{\alpha\beta}$ ($\alpha,\beta = L$; R) of the process as:

$$M_{i\to f} = \frac{e^2}{4}\{F_{LL}[\bar{\upsilon}(p_2)\gamma_\mu(1+\gamma_5)u(p_1)][\bar{u}(k_1)\gamma_\mu(1+\gamma_5)\upsilon(k_2)] + F_{LR}[\bar{\upsilon}(p_2)\gamma_\mu(1+\gamma_5)u(p_1)]\times$$
$$\times[\bar{u}(k_1)\gamma_\mu(1-\gamma_5)\upsilon(k_2)] + F_{RL}[\bar{\upsilon}(p_2)\gamma_\mu(1-\gamma_5)u(p_1)][\bar{u}(k_1)\gamma_\mu(1+\gamma_5)\upsilon(k_2)] +$$
$$+F_{RR}[\bar{\upsilon}(p_2)\gamma_\mu(1-\gamma_5)u(p_1)][\bar{u}(k_1)\gamma_\mu(1-\gamma_5)\upsilon(k_2)]\}, \qquad (23)$$

where

$$F_{\alpha\beta} = \frac{Q_\mu Q_f}{s} + D_Z(s)\frac{g_\alpha(\mu)g_\beta(f)}{x_W(1-x_W)} \quad (\alpha,\beta = L;R) \qquad (24)$$

spiral amplitudes; $D_Z(s) = [s - M_Z^2 + iM_Z\Gamma_Z]^{-1}$, Γ_Z – the full width of the Z -boson; $Q_\mu - 1(Q_f)$ –the electric charge of the muon (fermion); $g_L(\mu)$ ($g_L(f)$ and $g_R(\mu)$ ($g_R(f)$)–are the left and right constants of the interaction of the muon (fermion) with the vector Z -boson:

$$g_L(\mu) = -\frac{1}{2} + x_W, \qquad g_R(\mu) = x_W,$$
$$g_L(f) = I_3(f) - Q_f x_W, \quad g_R(f) = -Q_f x_W, \qquad (25)$$

$I_3(f)$ – the third projection of the weak fermion isospin, $x_W = \sin^2\theta_W$ – the Weinberg parameter.

The differential cross sections corresponding to the four spiral amplitudes are equal (there is no interference between different amplitudes due to the preservation of the helicities of the particles):

$$\frac{d\sigma}{d\Omega}(\mu_L^-\mu_R^+ \to f_L\bar{f}_R) = \frac{\alpha^2}{4}N_C s|F_{LL}|^2(1+\cos\theta)^2,$$
$$\frac{d\sigma}{d\Omega}(\mu_L^-\mu_R^+ \to f_R\bar{f}_L) = \frac{\alpha^2}{4}N_C s|F_{LR}|^2(1-\cos\theta)^2,$$
$$\frac{d\sigma}{d\Omega}(\mu_R^-\mu_L^+ \to f_L\bar{f}_R) = \frac{\alpha^2}{4}N_C s|F_{RL}|^2(1-\cos\theta)^2, \qquad (26)$$
$$\frac{d\sigma}{d\Omega}(\mu_R^-\mu_L^+ \to f_R\bar{f}_L) = \frac{\alpha^2}{4}N_C s|F_{RR}|^2(1+\cos\theta)^2,$$

where θ – is the angle between the directions of the fermion and muon impulses.

As follows from these spiral sections, at the zero angle of departure of the fermion ($\theta = 0$), the spiral sections of the processes $\mu_L^-\mu_R^+ \to f_R\bar{f}_L$ and $\mu_R^-\mu_L^+ \to f_L\bar{f}_R$, and at the angle of departure of the fermion $\theta = \pi$, the spiral sections of the processes $\mu_L^-\mu_R^+ \to f_L\bar{f}_R$ and $\mu_R^-\mu_L^+ \to f_R\bar{f}_L$ turn to zero. This fact is connected with the law of conservation of the total moment in spiral processes.

Fig. 8 shows the directions of the impulses and spins of the particles in the reaction $\mu^-\mu^+ \to (\gamma^*, Z^*) \to f\bar{f}$. As can be seen, the spin vectors of the initial particles are directed in one direction, therefore, their total spin moment is equal to one, the spin of the virtual photon and the Z -boson is also equal to one.

We denote by λ_1 and λ_2 (h_1 and h_2) the helicities of the muon and antimuon (fermion and antifermion). Then, in the case of longitudinally polarized initial and final particles, the differential cross-section of the process $\mu^-\mu^+ \to (\gamma^*, Z^*) \to f\bar{f}$ can be represented as:

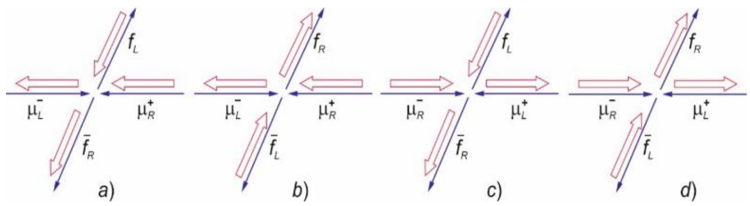

Fig. 8. Directions of impulses and spin vectors of particles in the reaction $\mu^-\mu^+ \to f\bar{f}$,

$$\frac{d\sigma}{d(\cos\theta)}(\lambda_1,\lambda_2,h_1,h_2) = \frac{\pi\alpha^2 N_C}{32}s\times$$
$$\{[|F_{LL}|^2(1-\lambda_1)(1+\lambda_2)(1-h_1)(1+h_2) + |F_{RR}|^2(1+\lambda_1)(1-\lambda_2)(1+h_1)(1-h_2)](1+\cos\theta)^2 +$$
$$+[|F_{LR}|^2(1-\lambda_1)(1+\lambda_2)(1+h_1)(1-h_2) + |F_{RL}|^2(1+\lambda_1)(1-\lambda_2)(1-h_1)(1+h_2)(1-\cos\theta)^2]\}. \qquad (27)$$

Thus, the contributions of diagrams a) and b) in fig. 1 to the process $\mu^-\mu^+ \to f\bar{f}$ cross-section differ significantly from each other in terms of the spiral properties of the muon-antimuon and fermion-antifermion pairs. Indeed, according to diagram a), the muon-antimuon of the pair, as well as the fermion-antifermion of the pair, must have the same helicities ($\mu_L^-\mu_L^+ \to f_L\bar{f}_L$, $\mu_L^-\mu_L^+ \to f_R\bar{f}_R$, $\mu_R^-\mu_R^+ \to f_L\bar{f}_L$,

$\mu_R^- \mu_R^+ \to f_R \bar{f}_R$). However, under this condition, the contribution of diagram b) to the cross-section of the process turns to zero. If the muon-antimuon pairs with a fermion-anti-fermion pore have opposite helicities ($\mu_L^- \mu_R^+ \to f_L \bar{f}_R$, $\mu_L^- \mu_R^+ \to f_R \bar{f}_L$, $\mu_R^- \mu_L^+ \to f_L \bar{f}_R$, $\mu_R^- \mu_L^+ \to f_R \bar{f}_L$), then the contribution of diagram a) to the cross section is zero. Consequently, the spiral properties of the muon-antimuon pairs make it possible to isolate the contributions of diagrams a) and b) to the cross-section of the process under consideration.

Let's move on to the discussion of the most observed characteristics of the process $\mu^- \mu^+ \to (\gamma^*, Z^*) \to f\bar{f}$. These characteristics are:

1. forward-backward asymmetry in the case of unpolarized particles

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} \cdot \frac{|F_{LL}|^2 + |F_{RR}|^2 - |F_{LR}|^2 - |F_{RL}|^2}{|F_{LL}|^2 + |F_{LR}|^2 + |F_{RL}|^2 + |F_{RR}|^2} \tag{28}$$

where σ_F and σ_B –is the effective cross-section of the fermion production in the interior and posterior hemispheres:

$$\sigma_F = \int_0^1 \frac{d\sigma}{d(\cos\theta)} d(\cos\theta) = \frac{\pi\alpha^2 N_C s}{24} = \{7[|F_{LL}|^2 + |F_{RR}|^2] + |F_{LR}|^2 + |F_{RL}|^2\},$$
$$\sigma_B = \int_{-1}^0 \frac{d\sigma}{d(\cos\theta)} d(\cos\theta) = \frac{\pi\alpha^2 N_C s}{24} = \{|F_{LL}|^2 + |F_{RR}|^2 + 7[|F_{LR}|^2 + |F_{RL}|^2]\}; \tag{29}$$

2. left-right spin asymmetry

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{|F_{LL}|^2 + |F_{LR}|^2 - |F_{RL}|^2 - |F_{RR}|^2}{|F_{LL}|^2 + |F_{LR}|^2 + |F_{RL}|^2 + |F_{RR}|^2}, \tag{30}$$

here σ_L and σ_R – are the effective cross-sections of the processes $\mu_L^- \mu^+ \to f\bar{f}$ and $\mu_R^- \mu^+ \to f\bar{f}$:

$$\sigma_L = \frac{2}{3}\pi\alpha^2 N_C s[|F_{LL}|^2 + |F_{LR}|^2],$$
$$\sigma_R = \frac{2}{3}\pi\alpha^2 N_C s[|F_{RL}|^2 + |F_{RR}|^2]; \tag{31}$$

3. the degree of longitudinal polarization of the fermion

$$P_f = \frac{\sigma(f_R) - \sigma(f_L)}{\sigma(f_R) + \sigma(f_L)} = \frac{|F_{LR}|^2 + |F_{RR}|^2 - |F_{LL}|^2 - |F_{RL}|^2}{|F_{LL}|^2 + |F_{RL}|^2 + |F_{LR}|^2 + |F_{RR}|^2}, \tag{32}$$

where $\sigma(f_R)$ and $\sigma(f_L)$ – are the integral effective cross sections of fermion f generation in the final state with experimentally measurable right and left polarization

$$\sigma(f_R) = \frac{1}{3}\pi\alpha^2 N_C s[|F_{RR}|^2 + |F_{LR}|^2],$$
$$\sigma(f_L) = \frac{1}{3}\pi\alpha^2 N_C s[|F_{RL}|^2 + |F_{LL}|^2]. \tag{33}$$

In the case when the muon-antimuon pair is transversely polarized, and summation is performed according to the polarization states of the fermion-antifermion pair, the differential cross-section of the process $\mu^- \mu^+ \to (\gamma^*, Z^*) \to f\bar{f}$ can be represented as follows:

$$\frac{d\sigma(\xi_1^\perp \xi_2^\perp)}{d\Omega} = \frac{\alpha^2 N_C}{16} s\{[|F_{LL}|^2 + |F_{RR}|^2](1+\cos\theta)^2 + [|F_{LR}|^2 + |F_{RL}|^2](1-\cos\theta)^2 -$$
$$- 2\xi_1^\perp \xi_2^\perp \sin^2\theta[\mathrm{Re}(F_{LL}F_{RL}^* + F_{LR}F_{RR}^*)\cos 2\varphi + \mathrm{Im}(F_{LL}F_{RL}^* + F_{LR}F_{RR}^*)\sin 2\varphi]\}, \tag{34}$$

Where φ – is the azimuthal angle of the fermion f departure, calculated from the plane of transverse polarization of the muon-antimuon pair.

The differential annihilation cross-section of a transversely polarized muon-antimuon pair (34) leads to the following transverse spin asymmetries:

$$A_{\varphi}^{(1)} = \frac{2}{\xi_1^{\perp}\xi_2^{\perp}} \frac{\int_0^{2\pi} \cos 2\varphi dg\varphi \int_{-1}^{1} d(\cos\theta)\left(\frac{d\sigma}{d\Omega}\right)}{\int_0^{2\pi} d\varphi \int_{-1}^{1} d(\cos\theta)\left(\frac{d\sigma}{d\Omega}\right)} = -\frac{\mathrm{Re}(F_{LL}F_{RL}^* + F_{LR}F_{RR}^*)}{|F_{LL}|^2 + |F_{LR}|^2 + |F_{RL}|^2 + |F_{RR}|^2}, \quad (35)$$

$$A_{\varphi}^{(2)} = \frac{2}{\xi_1^{\perp}\xi_2^{\perp}} \frac{\int_0^{2\pi} \sin 2\varphi dg\varphi \int_{-1}^{1} d(\cos\theta)\left(\frac{d\sigma}{d\Omega}\right)}{\int_0^{2\pi} d\varphi \int_{-1}^{1} d(\cos\theta)\left(\frac{d\sigma}{d\Omega}\right)} = -\frac{\mathrm{Im}(F_{LL}F_{RL}^* + F_{LR}F_{RR}^*)}{|F_{LL}|^2 + |F_{LR}|^2 + |F_{RL}|^2 + |F_{RR}|^2}. \quad (36)$$

We will estimate the above asymmetries in the reaction $\mu^-\mu^+ \to (\gamma^*, Z^*) \to \tau^-\tau^+$ at the value of the Weinberg parameter $x_W = 0.2315$, the mass of the $M_Z = 91.1875$ GeV Z-boson (the full width of the decay of the $\Gamma_Z = 2.4952$ GeV Z-boson is not taken into account, since $s - m_Z^2 >> M_Z\Gamma_Z$). Figure 9 illustrates the dependence of the asymmetry on the energy of a muon-antimuon pair. It can be seen that the forward-backward asymmetry A_{FB} and the left-right spin asymmetry are positive and decrease with increasing energy. The degree of longitudinal polarization of the τ^--lepton and the transverse spin asymmetry are negative and P_f increases with increasing energy $\sqrt{s}$, and A_{φ} decreases with increasing energy. The transverse spin asymmetry $A_{\varphi}^{(2)}$ does not occur, since the imaginary parts of the spiral amplitudes $F_{\alpha\beta}$ are not taken into account.

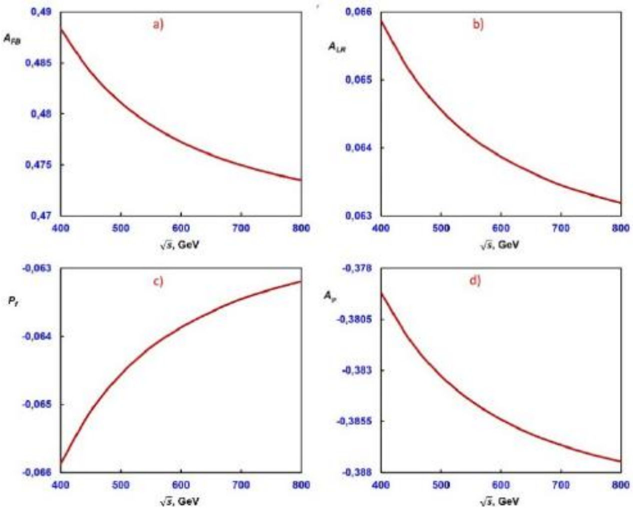

Fig. 9. Energy dependence of the asymmetry in the reaction $\mu^-\mu^+ \to (\gamma^*, Z^*) \to \tau^-\tau^+$.

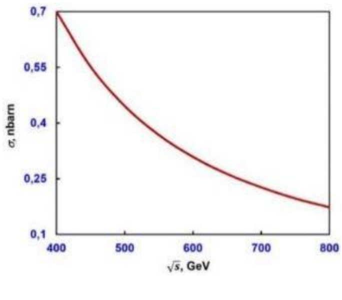

Fig. 10. Energy dependence of the reaction cross-section $\mu^-\mu^+ \to (\gamma^*, Z^*) \to \tau^-\tau^+$

In the case of unpolarized particles, the total effective cross-section of the reaction $\mu^-\mu^+ \to (\gamma^*, Z^*) \to f\bar{f}$ is determine by the formula

$$\sigma(\mu^-\mu^+ \to (\gamma^*, Z^*) \to f\bar{f}) = \frac{\pi\alpha^2}{3} N_C s[|F_{LL}|^2 + |F_{LR}|^2 + |F_{RL}|^2 + |F_{RR}|^2]. \quad (37)$$

Fig. 10 shows the dependence of the effective cross-section of the reaction $\mu^-\mu^+ \to (\gamma^*, Z^*) \to \tau^-\tau^+$ on the energy of the muon-antimuon pair. As can be seen from the figure, with increasing energy $\sqrt{s}$, the effective cross-section of the reaction decreases.

7. EFFECTIVE CROSS-SECTION OF REACTION $\mu^-\mu^+ \to (\gamma^*, Z^*) \to t\bar{t}$

In the process of creating a heavy top quark pair during the annihilation of a muon-antimuon pair, we cannot neglect the mass of the top quark. The amplitude of this reaction can be write as follows:

$$M_\gamma + M_Z = \frac{e^2}{s} Q_\mu Q_t [\bar{\upsilon}(p_2,s_2)\gamma_\mu u(p_1,s_1)][\bar{u}(k_1,r_1)\gamma_\mu \upsilon(k_2,r_2)] +$$
$$+ \frac{e^2}{4x_W(1-x_W)} \cdot \frac{1}{s-M_Z^2}[\bar{\upsilon}(p_2,s_2)\gamma_\mu(g_L(\mu)(1+\gamma_5)+g_R(\mu)(1-\gamma_5))u(p_1,s_1)]\times$$
$$\times[\bar{u}(k_1,r_1)\gamma_\mu(g_L(t)(1+\gamma_5)+g_R(t)(1-\gamma_5))\upsilon(k_2,r_2)]. \quad (38)$$

The square of the amplitude of this process is expressed by muon and top-quark tensors:

$$\left|M_\gamma + M_Z\right|^2 = \left|M_\gamma\right|^2 + \left|M_Z\right|^2 + (M_\gamma^* M_Z + M_Z^* M_\gamma), \ \left|M_\gamma\right|^2 = \frac{e^4}{s^2}(Q_\mu Q_t)^2 \cdot L_{\mu\nu}^{(\gamma)} T_{\mu\nu}^{(\gamma)},$$

$$\left|M_Z\right|^2 = \frac{e^4}{16x_W^2(1-x_W)^2} \cdot \frac{1}{(s-M_Z^2)^2} \cdot L_{\mu\nu}^{(Z)} T_{\mu\nu}^{(Z)}, \ M_\gamma^* M_Z + M_Z^* M_\gamma = \frac{2e^4}{4x_W(1-x_W)} \cdot \frac{Q_\mu Q_t}{s(s-M_Z^2)} \cdot L_{\mu\nu}^{(I)} T_{\mu\nu}^{(I)}.$$

Electromagnetic $L_{\mu\nu}^{(\gamma)}$ and $T_{\mu\nu}^{(\gamma)}$, weak $L_{\mu\nu}^{(Z)}$ and $T_{\mu\nu}^{(Z)}$, as well as interference tensors $L_{\mu\nu}^{(I)}$ and $T_{\mu\nu}^{(I)}$, including the product of these tensors, are given in the Appendix. Here we present the cross sections of the process $\mu^-\mu^+ \to (\gamma^*, Z^*) \to t\bar{t}$ in the center of mass system at certain values of the helicities of the initial and final particles. Using the products of the muon and top-quark tensors given in the Appendix, we find the following expression for the effective cross section

$$\frac{d\sigma}{d\Omega}(\mu^-\mu^+ \to (\gamma^*, Z^*) \to t\bar{t}) = \frac{\alpha^2 N_C}{64s}\sqrt{1-\frac{4m_t^2}{s}} \times$$
$$\times\left\{4(Q_\mu Q_t)^2\left[(1-\lambda_1\lambda_2)\left((1-h_1h_2)(1+\cos^2\theta)+(1+h_1h_2)\cdot\frac{4m_t^2}{s}\cdot\sin^2\theta\right)+2(\lambda_2-\lambda_1)(h_2-h_1)\cos\theta\right]+\right.$$
$$+\frac{s^2}{x_W^2(1-x_W)^2}\cdot\frac{1}{(s-M_Z^2)^2}\left[((g_L^2(\mu)+g_R^2(\mu)(1-\lambda_1\lambda_2)+(g_L^2(\mu)-g_R^2(\mu))(\lambda_2-\lambda_1)))\times\right.$$
$$\times\left((g_L^2(t)+g_R^2(t))\left((1-h_1h_2)(1+\cos^2\theta)+(1+h_1h_2)\cdot\frac{4m_t^2}{s}\cdot\sin^2\theta\right)+(g_L^2(t)-g_R^2(t))\sqrt{1-\frac{4m_t^2}{s}}\times\right.$$
$$\left.\times(h_2-h_1)(1+\cos^2\theta)+2g_L(t)g_R(t)\cdot\frac{4m_t^2}{s}(1+h_1h_2)\sin^2\theta)\right)+\left((g_L^2(\mu)-g_R^2(\mu)(1-\lambda_1\lambda_2)+\right.$$
$$\left.+(g_L^2(\mu)+g_R^2(\mu))(\lambda_2-\lambda_1)\right)\left((g_L^2(t)+g_R^2(t))(h_2-h_1)\left(1-\frac{2m_t^2}{s}\right)+(g_L^2(t)-g_R^2(t))2\sqrt{1-\frac{4m_t^2}{s}}(1-h_1h_2)+\right.$$
$$\left.\left.\left.+2g_L(t)g_R(t)\frac{4m_t^2}{s}(h_2-h_1)\right)\cos\theta\right)\right]+\frac{2Q_eQ_t}{x_W(1-x_W)}\cdot\frac{s}{(s-M_Z^2)}[((g_L(\mu)+g_R(\mu)(1-\lambda_1\lambda_2)+$$
$$+((g_L(\mu)-g_R(\mu))(\lambda_2-\lambda_1))\left[(g_L(t)+g_R(H)((1-h_1h_2)(1+\cos^2\theta)+\frac{4m_t^2}{s}(1+h_1h_2)\sin^2\theta)+\right.$$
$$\left.+(g_L(t)-g_R(t))\sqrt{1-\frac{4m_t^2}{s}}(h_2-h_1)(1+\cos^2\theta)\right]+((g_L(\mu)-g_R(\mu))(1-\lambda_1\lambda_2)+$$
$$+((g_L(\mu)+g_R(\mu))(\lambda_2-\lambda_1)[(g_L(t)+g_R(t))(h_2-h_1)\left(1+\frac{2m_t^2}{s}\right)\cos\theta+$$
$$\left.\left.+((g_L(t)-g_R(t))\cdot 2\cdot(1-h_1h_2)\sqrt{1-\frac{4m_t^2}{s}}\cos\theta\right]\right]\right\}. \quad (39)$$

We have neglected the mass of the muon compared to its energy ($m_\mu^2 << s$), as a result, the helicity of the muon and the antimuon are opposite: $\lambda_1 = -\lambda_2 = \pm 1$. However, we have left the mass of the top quark, while the top quark and the top antiquark must have the same helicities in the mass terms $(\sim m_t^2/s)$: $h_1 = h_2 = \pm 1$.

Due to the consideration of the mass of the top quark, the process $\mu^-\mu^+ \to t\bar{t}$ is characterize by spiral sections:

$$\frac{d\sigma}{d(\cos\theta)}(\mu_L^-\mu_R^+ \to t_R\bar{t}_R) = \frac{3\pi\alpha^2 s}{8}\sqrt{1-\frac{4m_t^2}{s}}[F_{LL}+F_{LR}]^2\cdot\frac{4m_t^2}{s}\sin^2\theta,$$

$$\frac{d\sigma}{d(\cos\theta)}(\mu_L^-\mu_R^+ \to t_L\bar{t}_L) = \frac{3\pi\alpha^2 s}{8}\sqrt{1-\frac{4m_t^2}{s}}[F_{LL}+F_{LR}]^2\cdot\frac{4m_t^2}{s}\sin^2\theta,$$

$$\frac{d\sigma}{d(\cos\theta)}(\mu_L^-\mu_R^+ \to t_L\bar{t}_R) = \frac{3\pi\alpha^2 s}{8}\sqrt{1-\frac{4m_t^2}{s}}\left[F_{LL}\left(1+\sqrt{1-\frac{4m_t^2}{s}}\right)+F_{LR}\left(1-\sqrt{1-\frac{4m_t^2}{2}}\right)\right]^2(1+\cos\theta)^2, \quad (40)$$

$$\frac{d\sigma}{d(\cos\theta)}(\mu_L^-\mu_R^+ \to t_R\bar{t}_L) = \frac{3\pi\alpha^2 s}{8}\sqrt{1-\frac{4m_t^2}{s}}\left[F_{LL}\left(1-\sqrt{1-\frac{4m_t^2}{s}}\right)+F_{LR}\left(1+\sqrt{1-\frac{4m_t^2}{2}}\right)\right]^2(1-\cos\theta)^2.$$

We will obtain similar spiral flows during the annihilation of a right-polarized muon and a left-polarized anti-muon: $\mu_R^-\mu_L^+ \to t\bar{t}$. To obtain the spiral sections of these processes, it is necessary to make changes in the spiral amplitudes of the following substitutions: $L \leftrightarrow R$. If no information about the spins of the particles is recorded in the experiments, we must perform averaging over the spins of the incoming particles and summing over the spins of the particles in the final state. For the cross section of the process $\mu^-\mu^+ \to (\gamma^*, Z^*) \to t\bar{t}$ in the case of unpolarized initial and final particles, we find:

$$\frac{d\sigma_0}{d\Omega}(\mu^-\mu^+ \to (\gamma^*,Z^*) \to t\bar{t}) = \frac{\alpha^2 N_C}{16s}\sqrt{1-\frac{4m_t^2}{s}}\times$$
$$\times\Bigg\{4(Q_\mu Q_t)^2\left(1+\cos^2\theta+\frac{4m_t^2}{s}\cdot\sin^2\theta\right)+\frac{s^2}{(s-M_Z^2)^2}\cdot\frac{1}{x_W^2(1-x_W)^2}\times$$
$$\times\Bigg[(g_L^2(\mu)+g_R^2(\mu))\left((g_L^2(t)+g_R^2(t))\left(1+\cos^2\theta+\frac{4m_t^2}{s}\cdot\sin^2\theta\right)+2g_L(t)g_R(t)\frac{4m_t^2}{s}\right)-$$
$$-2(g_L^2(\mu)-g_R^2(\mu))(g_L^2(t)-g_R^2(t))\sqrt{1-\frac{4m_t^2}{s}}\cos\theta\Bigg]+$$
$$+\frac{s}{s-M_Z^2}\cdot\frac{2Q_\mu Q_t}{x_W(1-x_W)}\Bigg[(g_L(\mu)+g_R(\mu))(g_L(t)+g_R(t))\left[1+\cos^2\theta+\frac{4m_t^2}{s}\cdot\sin^2\theta\right]+$$
$$+2[g_L(\mu)-g_R(\mu)][g_L(t)-g_R(t)]\sqrt{1-\frac{4m_t^2}{s}}\cos\theta\Bigg]\Bigg\}. \quad (41)$$

Fig. 11 shows the dependence of the differential effective cross-section of the reaction $\mu^-\mu^+ \to (\gamma^*, Z^*) \to t\bar{t}$ on the angle θ at $\sqrt{s} = 500$ GeV, $m_t = 173.2$ GeV and $x_W = 0.2315$. As can be seen from the figure, with an increase in the departure angle of the top quark, the effective cross-section of the reaction $\mu^-\mu^+ \to t\bar{t}$ decreases. I integrate the cross-section (41) at the angles of departure of the top quark θ and φ, we obtain an expression for the integral cross-section of the process in the following form:

$$\sigma_0(\mu^-\mu^+ \to (\gamma^*,Z^*) \to t\bar{t}) = \frac{\pi\alpha^2 N_C}{4s}\Bigg\{4(Q_\mu Q_t)^2\cdot\frac{4}{3}\left(1+\frac{2m_t^2}{s}\right)+\frac{s^2}{(s-M_Z^2)^2}\cdot\frac{1}{x_W^2(1-x_W)^2}\times$$
$$\times\left[(g_L^2(\mu)+g_R^2(\mu))\left((g_L^2(t)+g_R^2(t))\frac{4}{3}\left(1+\frac{2m_t^2}{s}\right)+2g_L(t)g_R(t)\cdot\frac{4m_t^2}{s}\right)\right]+\frac{s}{s-M_Z^2}\cdot\frac{2Q_\mu Q_t}{x_W(1-x_W)}\times$$
$$\times[g_L(\mu)+g_R(\mu)][g_L(t)+g_R(t)]\cdot\frac{4}{3}\cdot\left(1+\frac{2m_t^2}{s}\right)\Bigg\}. \quad (42)$$

Fig. 12 illustrates the dependence of the total cross-section of the reaction $\mu^-\mu^+ \to (\gamma^*, Z^*) \to t\bar{t}$ on the energy of the muon-antimuon pair $\sqrt{s}$. It is observed that with increasing energy, the cross-section of the process decreases.

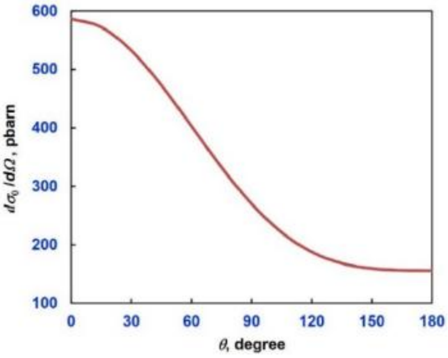

Fig. 11. Angular dependence of the reaction cross-section $\mu^-\mu^+ \to t\bar{t}$.

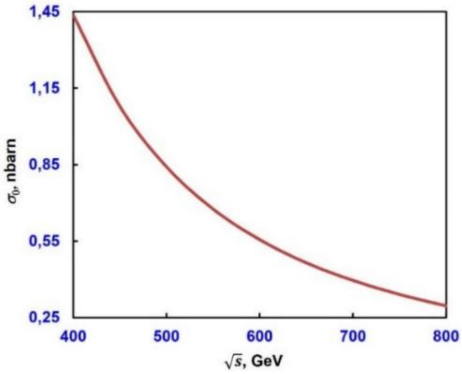

Fig. 12. Energy dependence of the process cross-section $\mu^-\mu^+ \to t\bar{t}$

CONCLUSION

In the framework of the Minimal Supersymmetric Standard Model, we discussed the process of annihilation of muon-antimuon pairs into a fermion-antifermion pair: $\mu^-\mu^+ \to f\bar{f}$. Taking into account the arbitrary polarizations of the initial and final particles at the same time, a general expression for the effective cross-section of the process is obtained. Diagrams with the exchange of Higgs bosons H^*, h^* and A^*, as well as with the exchange of a photon γ^* and a neutral Z^*-boson, are studied in detail.

Expressions for longitudinal and transverse spin asymmetries, as well as the degrees of longitudinal and transverse polarizations of the tau-lepton and top quark are determined, the study of which is a source of information about the physical nature of the Higgs bosons H , h and A .

APPLICATION

Here we give the expressions of the muon (top quark) tensors $L^{(\gamma)}_{\mu\nu}$, $L^{(Z)}_{\mu\nu}$, $L^{(I)}_{\mu\nu}$ ($T^{(\gamma)}_{\mu\nu}$, $T^{(Z)}_{\mu\nu}$, $T^{(I)}_{\mu\nu}$) and the products of these tensors (we assume that the muon and anti-muon are longitudinally polarized)

$$L^{(\gamma)}_{\mu\nu} = (1-\lambda_1\lambda_2)[p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu} - (p_1\cdot p_2)g_{\mu\nu}] - i(\lambda_2-\lambda_1)(\mu\nu p_1 p_2)_\varepsilon ,$$

$$L^{(Z)}_{\mu\nu} = 2[g_L^2(\mu)+g_R^2(\mu)]L^{(\gamma)}_{\mu\nu} + 2[g_L^2(\mu)-g_R^2(\mu)]\{(\lambda_2-\lambda_1)[p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu} - (p_1\cdot p_2)g_{\mu\nu}] + i(1-\lambda_1\lambda_2)(\mu\nu p_2 p_1)_\varepsilon\},$$

$$L^{(I)}_{\mu\nu} = [g_L(\mu)+g_R(\mu)]L^{(\gamma)}_{\mu\nu} + [g_L(\mu)-g_R(\mu)]\{(\lambda_2-\lambda_1)[p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu} - (p_1\cdot p_2)g_{\mu\nu}] + (1-\lambda_1\lambda_2)(-i)(\mu\nu p_1 p_2)_\varepsilon\},$$

$$T^{(\gamma)}_{\mu\nu} = k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu} - (k_1\cdot k_2)g_{\mu\nu} - m_t^2[r_{1\mu}r_{2\nu} + r_{2\mu}r_{1\nu} - (r_1\cdot r_2)g_{\mu\nu}] + im_t[(\mu\nu k_1 r_2)_\varepsilon + (\mu\nu k_2 r_1)_\nu +$$
$$+ (\mu\nu k_2 r_2)_\varepsilon + (\mu\nu r_1 k_1)_\varepsilon] - m_t^2 g_{\mu\nu} - (k_1\cdot k_2)[r_{1\mu}r_{2\nu} + r_{2\mu}r_{1\nu} - (r_1\cdot r_2)g_{\mu\nu}] + (k_1\cdot r_2)[k_{2\mu}r_{1\nu} + k_{2\nu}r_{2\mu} - (k_2\cdot r_1)g_{\mu\nu}] -$$
$$-(r_1\cdot r_2)[k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu}] + (k_2\cdot r_1)[k_{1\mu}r_{2\nu} + k_{1\nu}r_{2\mu}],$$

$$T^{(Z)}_{\mu\nu} = 2[g_L^2(t)+g_R^2(t)]\{k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu} - (k_1\cdot k_2)g_{\mu\nu} - m_t^2[r_{1\mu}r_{2\nu} + r_{2\mu}r_{1\nu} - (r_1\cdot r_2)g_{\mu\nu}] +$$
$$+ im_t[(\mu\nu k_1 r_2)_\varepsilon - (\mu\nu k_2 r_1)_\varepsilon] + 2[g_L^2(t)-g_R^2(t)]\}\{m_t[k_{1\mu}r_{2\nu} + r_{2\mu}k_{1\nu} - (k_1\cdot r_2)g_{\mu\nu} - r_{1\mu}k_{2\nu} - r_{1\nu}k_{2\mu} + (r_1\cdot k_2)g_{\mu\nu}] +$$
$$i(\mu\nu k_1 k_2)_\varepsilon - im_t^2(\mu\nu r_1 r_2)_\varepsilon\} + 4g_L(t)g_R(t)\{-m_t^2 g_{\mu\nu} + (k_1\cdot k_2)[r_{1\mu}r_{2\nu} + r_{2\mu}r_{1\nu} - (r_1\cdot r_2)g_{\mu\nu}] -$$
$$-(k_1\cdot r_2)[r_{1\mu}k_{2\nu} + r_{1\nu}k_{2\mu} - (r_1\cdot k_2)g_{\mu\nu}] + (r_1\cdot r_2)[k_{1\mu}r_{2\nu} + k_{2\mu}k_{1\nu}] - (k_2\cdot r_1)[k_{1\mu}r_{2\nu} + k_{1\nu}r_{2\mu}]\},$$

$$T^{(I)}_{\mu\nu} = [g_L(t)+g_R(t)]\{k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu} - (k_1\cdot k_2)g_{\mu\nu} - m_t^2[r_{1\mu}r_{2\nu} + r_{2\mu}r_{1\nu} - (r_1\cdot r_2)g_{\mu\nu}] - m_t^2 g_{\mu\nu} - (k_1\cdot k_2)[r_{1\mu}r_{2\nu} +$$
$$+ r_{1\nu}r_{2\mu} - (r_1\cdot r_2)g_{\mu\nu}] + (k_1\cdot r_2)[r_{1\mu}k_{2\nu} + r_{1\nu}k_{2\mu}] - (r_1\cdot r_2)[k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu}] + (k_2\cdot r_1)[k_{1\mu}r_{2\nu} + k_{1\nu}r_{2\mu}]\},$$

$$L_{\mu\nu}^{(\gamma)}T_{\mu\nu}^{(\gamma)}=2(1-\lambda_1\lambda_2)[(p_1\cdot k_1)(p_2\cdot k_2)+(p_1\cdot k_2)(p_2\cdot k_1)-m_t^2(p_1\cdot r_1)(p_2\cdot r_2)+(p_1\cdot r_2)(p_2\cdot r_1)+m_t^2(p_1\cdot p_2)-$$
$$-(k_1\cdot k_2)[(p_1\cdot r_1)(p_2\cdot r_2)+(p_1\cdot r_2)(p_2\cdot r_1)]+(k_1\cdot r_2)[(p_2\cdot k_2)(p_1\cdot r_1)+(p_1\cdot k_2)(p_2\cdot r_1)]-$$
$$-(r_1\cdot r_2)[(p_1\cdot k_1)(p_2\cdot k_2)+(p_1\cdot k_2)(p_2\cdot k_1)-(p_1\cdot p_2)(k_1\cdot k_2)]+(k_2\cdot r_1)[(p_1\cdot k_1)(p_2\cdot r_2)+$$
$$+(p_2\cdot k_1)(p_1\cdot r_2)-(p_1\cdot p_2)(k_1\cdot r_2)]+2(\lambda_2-\lambda_1)m_t[(p_2\cdot k_1)(p_1\cdot r_2)-(p_1\cdot k_1)(p_2\cdot r_2)+(p_2\cdot k_2)(p_1\cdot r_1)-$$
$$-(p_1\cdot k_2)(p_2\cdot r_1)+(p_2\cdot k_2)(p_1\cdot r_2)-(p_1\cdot k_2)(p_2\cdot r_2)+(p_1\cdot k_1)(p_2\cdot r_1)-(p_2\cdot k_1)(p_1\cdot r_1)],$$

$$L_{\mu\nu}^{(Z)}T_{\mu\nu}^{(Z)}=8\{[g_L^2(\mu)+g_R^2(\mu)](1-\lambda_1\lambda_2)+[g_L^2(\mu)-g_R^2(\mu)](\lambda_2-\lambda_1)]\}\{[g_L^2(t)+g_R^2(t)][(p_1\cdot k_1)(p_2\cdot k_2)+$$
$$+(p_1\cdot k_2)(p_2\cdot k_1)-m_t^2((p_1\cdot r_1)(p_2\cdot r_2)+(p_1\cdot r_2)(p_2\cdot r_1))]+[g_L^2(t)-g_R^2(t)]m_t[(p_1\cdot k_1)(p_2\cdot r_2)+$$
$$+(p_1\cdot r_2)(p_2\cdot k_1)-(p_1\cdot r_1)(p_2\cdot k_2)-(p_2\cdot r_1)(p_1\cdot k_2)]+2g_L(t)g_R(t)[m_t^2(p_1\cdot p_2)+$$
$$+(k_1\cdot k_2)((p_1\cdot r_1)(p_2\cdot r_2)+(p_1\cdot r_2)(p_2\cdot r_1))-(k_1\cdot r_2)((p_1\cdot r_1)(p_2\cdot k_2)+(p_2\cdot r_1)(p_1\cdot k_2))+$$
$$+(r_1\cdot r_2)((p_1\cdot k_1)(p_2\cdot k_2)+(p_1\cdot k_2)(p_2\cdot k_1)-(p_1\cdot p_2)(k_1\cdot k_2))-(k_2\cdot r_1)((p_1\cdot k_1)(p_2\cdot r_2)+$$
$$+(p_2\cdot k_1)(p_1\cdot r_2)-(p_1\cdot p_2)(k_1\cdot r_2))]\}+8\{[g_L^2(\mu)-g_R^2(\mu)](1-\lambda_1\lambda_2)+[g_L^2(\mu)+g_R^2(\mu)](\lambda_2-\lambda_1)\}\times$$
$$\times\{[g_L^2(t)+g_R^2(t)]m_t[(p_1\cdot r_2)(p_2\cdot k_1)-(p_1\cdot k_1)(p_2\cdot r_2)+(p_1\cdot r_1)(p_2\cdot k_2)-(p_1\cdot k_2)(p_2\cdot r_1)]+$$
$$+[g_L^2(t)-g_R^2(t)][(p_1\cdot k_2)(p_2\cdot k_1)-(p_1\cdot k_1)(p_2\cdot k_2)+m_t^2(p_1\cdot r_1)(p_2\cdot r_2)-(p_1\cdot r_2)(p_2\cdot r_1)]+$$
$$+2g_L(t)g_R(t)m_t[(p_1\cdot r_2)(p_2\cdot k_2)-(p_2\cdot r_2)(p_1\cdot k_2)+(p_1\cdot r_1)(p_2\cdot k_1)-(p_2\cdot r_1)(p_1\cdot k_1)]\},$$

$$L_{\mu\nu}^{(I)}T_{\mu\nu}^{(I)}=2\{[g_L(\mu)+g_R(\mu)](1-\lambda_1\lambda_2)+[g_L(\mu)-g_R(\mu)](\lambda_2-\lambda_1)\}\{[g_L(t)+g_R(t)][(p_1\cdot k_1)(p_2\cdot k_2)+$$
$$+(p_1\cdot k_2)(p_2\cdot k_1)-m_t^2((p_1\cdot r_1)(p_2\cdot r_2)+(p_1\cdot r_2)(p_2\cdot r_1))]+[g_L(t)-g_R(t)]m_t[(p_1\cdot k_1)(p_2\cdot r_2)+$$
$$+(p_2\cdot k_1)(p_1\cdot r_2)-(p_2\cdot k_2)(p_1\cdot r_1)-(p_1\cdot k_2)(p_2\cdot r_1)]+[g_L(t)+g_R(t)][m_t^2(p_1\cdot p_2)-(k_1\cdot k_2)((p_1\cdot r_1)(p_2\cdot r_2)+$$
$$+(p_1\cdot r_2)(p_2\cdot r_1))+(k_1\cdot r_2)((p_2\cdot k_2)(p_1\cdot r_1)+(p_1\cdot k_2)(p_2\cdot r_1))-(r_1\cdot r_2)((p_1\cdot k_1)(p_2\cdot k_2)+(p_1\cdot k_2)(p_2\cdot k_1)-$$
$$-(p_1\cdot p_2)(k_1\cdot k_2))+(k_2\cdot r_1)((p_1\cdot k_1)(p_2\cdot r_2)+(p_2\cdot k_1)(p_1\cdot r_2)-(p_1\cdot p_2)(k_1\cdot r_2))]\}+2\{[g_L(\mu)-g_R(\mu)]\times$$
$$\times(1-\lambda_1\lambda_2)+[g_L(\mu)+g_R(\mu)](\lambda_2-\lambda_1)\}\{[g_L(t)+g_R(t)]m_t[(p_2\cdot k_1)(p_1\cdot r_2)-(p_1\cdot k_1)(p_2\cdot r_2)+$$
$$+(p_2\cdot k_2)(p_1\cdot r_1)-(p_1\cdot k_2)(p_2\cdot r_1)]\}$$

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[1] ATLAS Collaboration. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC // Phys. Letters, 2012, B716, p. 1-29.

[2] CMS Collaboration. Observation of a new boson at mass of 125 GeV with the CMS experiment at the LHC // Phys. Lett., 2012, B716, p.30-60.

[3] *V.A. Rubakov.* On Large Hadron Colliders discovery of a new particle with Higgs Boson properties // UFN, 2012, v.182, No 10, p.1017-1025 (in Russian).

[4] *A.V. Lanev.* CMS Collaboration results: Higgs boson and search for new physics // UFN, 2014, v. 184, №9, p. 996-1004 (In Russian).

[5] *D.I. Kazakov.* The Higgs boson is found: what is next? // UFN, 2014, v. 184, №9, p. 1004-1017 (In Russian).

[6] *P.W. Higgs.* Broken symmetries, massless particles and gauge fields // Phys. Lett., 1964, v. 12, p.132.

[7] *P.W. Higgs.* Broken symmetries and the masses of gauge bosons // Phys. Rev. Lett., 1964, v. 13, №16, p.508.

[8] *F. Englert, F. Brout.* Broken symmetry and the mass of gauge vector mesons // Phys. Rev. Lett., 1964, v. 13, №9, p.321.

[9] *A. Djouadi.* The Anatomy of Electro-Weak Symmetry Breaking. Tome II: The Higgs boson in the Minimal Supersymmetric Standard Model. arXiv: hep-ph /0503172v2, 2003; DOI: 10.1016/j.physrep. 2007.10.004.

[10] *J.F. Gunion, H.E. Haber.* Higgs bosons in Supersymmetric models (I) // Nucl. Phys., 1986, v. B272, p. 1-76.

[11] *H.E. Haber, G. Kane.* The search for supersymmetry: Probing physics beyond the Standard Model // Phys. Rep., 1985, v. C117, №2-4, p. 75-263.

[12] *J.F. Gunion.* Physics ata Muon Collider. arXiv: hep-ph/9802258, 1998, p. 23.

[13] *V.D. Shiltsev.* High energy particle colliders: past 20 years, next 20 years and beyond // UFN, 2012, v. 182, № 10, p. 1033-1046 (in Russian).

[14] *K. Peters.* Prospects for beyond Standard Model Higgs boson searches at future LHC runs and other machines. arXiv:1701.05124v.2 [hep-ex], 21 Feb. 2017, p. 9.

[15] *C.Blöchinger, M. Carena, J.Ellis. et al.* Physics Opportunities at $\mu^-\mu^+$ Higgs Factories. Report of the Higgs factory working group of the ECFA-CERN Study on Neutrino Factory, Muon Storage Rings at CERN.

[16] "ILC Reference Design Report", ILC-Report-2007-001; http://www.linearcollider.org.

[17] *A.A. Sokolov, I.M. Ternov.* Relyativistskiy electron, Moskva, "Nauka", 1974, p.392.

[18] *C. Partignani. et al.* Review of Particle Physics // Chinese Physics, 2016, v. C40, №10.

[19] *S.F. Hamilton*. Measurement of the longitudinal polarization of the top-quark in top-antitop events using the ATLAS detector. CERN-THESIS – 2014-008.

[20] *S.K. Abdullayev*. Standard Model, properties of leptons and quarks. Baku, Zeka Print, 2017, 276s.

[21] *S.K. Abdullayev, A.I. Mukhtarov*. Superstring Z'-boson in $e^{+}e^{-}$-annihilation // Phys. Part. Nucl., 1995, v. 26, № 5, p. 527-552.

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