EXCITATION OF UNSTABLE WAVES IN MULTI-VALLEY SEMICONDUCTORS GaAs TYPE IN EXTERNAL ELECTRIC AND STRONG MAGNETIC FIELDS

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Using the Boltzmann kinetic equation, the frequency of excited waves in two-valley semiconductors of the GaAs type in external electric and strong magnetic fields ($\mu H > c$) is theoretically calculated. Analytical expressions are found for the critical electric field at which an unstable electromagnetic wave is excited. A characteristic expression for the magnetic field H_{char} is found. The oscillation frequency of the electric field is calculated in three cases: 1) $H = H_{char}$, 2) $H > H_{char}$, 3) $H < H_{char}$. It is found that the frequency of the excited waves has the highest value in the case of $H > H_{char}$. It has been proven that the geometry of the sample L_x, L_y, L_z must be determined when unstable waves are excited inside the sample. The ratios between the sizes L_x, L_y, L_z are found. The directions of the external electric and magnetic fields significantly affect the frequencies of the excited waves. The theoretical calculation was carried out at $\vec{E}_0 \parallel \vec{H}_0$.

Keywords: effect Gunn's, Boltzmann equation, multi-valley semiconductors, external electric field, magnetic field, increment, frequency.
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INTRODUCTION

In theoretical studies [1–5], the excited oscillation of unstable waves was calculated inside a two-valley semiconductor of the GaAs type. These theoretical works were done when the external electric field is strong, i.e. $\vartheta_{дреф} > \vartheta_{зв}$. This criterion is satisfied if the external electric field is $E_0 > \frac{\vartheta_{зв}}{\mu_{под}}$ ($\mu_{под}$ -mobility of charge carriers). The external magnetic field changes as $\mu H > c$.

In the above works, a theoretical study was carried out with the fulfillment of the condition

$$\frac{dj}{dE} = \sigma_d = 0 \tag{1}$$

(j is the current flux density, E is the electric field, σ_d is the differential conductivity). However, from condition (1) it is impossible to determine the oscillation frequencies inside the sample. Therefore, to calculate the oscillation frequency of the excited wave inside the sample, we proceed as follows. First, the current density is calculated by applying the Boltzmann kinetic equation. After that, we calculate the current density using the Maxwell equation

$$\frac{\partial \vec{H}'}{\partial t} = -crot\vec{E}' \tag{2}$$

The current density

$$\vec{j} = \sigma\vec{E} + \sigma_1\left[\vec{E}\vec{H}\right] + \sigma_2\vec{H}\left[\vec{E}\vec{H}\right] \tag{3}$$

At condition

$$\vec{E}_0 = \vec{h}E_0 , \quad \vec{H}_0 = \vec{h}H_0 \tag{4}$$

This task is quite capable of determining the oscillation frequency of the excited waves inside the sample.

THEORY

Figure 1 shows the dependence of the current density in a spatially homogeneous system on the field strength. Here, the volt-ampere characteristics has a falling section and in the region $j_2 < j < j_p$, the field strength is a multivalued function of the current density. In this region, the system is in one of three states. Since the Gunn effect is associated with an N-shaped characteristic, electrical domains appear with negative differential conductivity. Domains appear by the Ridley-Watkins-Hilsum mechanism [5, 6]. Figure 2 shows the dispersion law in GaAs.

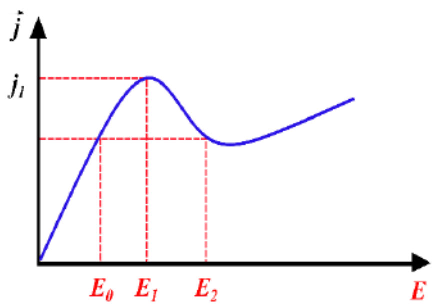

Fig.1. Dependence of the current density on the electric field in two-valley semiconductors of the GaAs type N-shaped characteristic.

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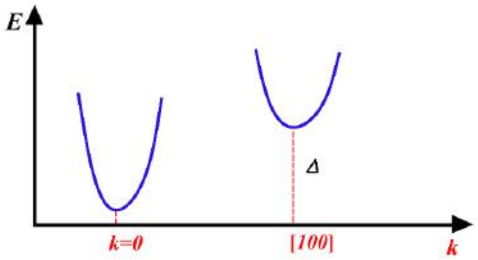

Fig.2. Dependence of the electron energy on the wave vector in GaAs.

The energy distance between the minimum $\Delta = 0.36eV$, $\Delta >> T_p$ is the lattice temperature. The presence of upper minimum does not affect the electron statistics. At a sufficiently high temperature, the electrons go to the upper minimum. Effective mass of electrons

$$m_a << m_b \tag{5}$$

m_a, $m_{б}$- effective mass of electrons in the lower and upper valleys, respectively.

Electron mobility

$$\mu_a << \mu_b \tag{6}$$

Then

$$\vec j = en_a\mu_a\vec E + en_b\mu_b\vec E \tag{7}$$

electron concentration

$$n = n_a + n_b = const \tag{8}$$

$eEl >> k_0T$ (e is elementary charge, l is electron mean free path). Diffusion current is neglected. Intervalley scattering is small compared to intravalley scattering. When solving the Boltzmann equation, the conditions for the appearance of current fluctuations were obtained.

Until now, there are no theoretical works of the Gunn effect, which take into account the intervalley scattering of the application of the Boltzmann kinetic equation.

In this paper, we will analyze the effect of a strong magnetic field on the Gunn effect, taking into account the above.

BASIC EQUATIONS OF THE PROBLEM

The state of charge carriers, described by the distribution function $f(\vec k, \vec r)$, which is the probability of electrons with a wave vector $\vec k$ ($\hbar\vec k$ is quasi-momentum) located near the point $\vec r$, is found from the Boltzmann kinetic equation. In a stationary process, the distribution function $f(\vec k, \vec r)$ does not depend on time, but under the influence of lattice vibrations (phonons) and crystal defects and under the influence of external factors, it changes, and these factors mutually compensate each other.

$$\left(\frac{\partial f}{\partial t}\right)_{exter} + \left(\frac{\partial f}{\partial t}\right)_{coll} = 0 \tag{9}$$

In the presence of external electric and magnetic fields, equation (9) has the form [7]

$$\vartheta\nabla_{\vec r} f + \frac{e}{h}\left\{\vec E + \frac{1}{c}\left[\vec\vartheta\vec H\right]\right\}\nabla_{\vec k} f = \left(\frac{\partial f}{\partial t}\right)_{coll} \tag{10}$$

Where $\vec\vartheta = \frac{1}{\hbar}\nabla_k\varepsilon(\vec k)$, ϑ is the electron velocity, $\nabla_{\vec r}$ and $\nabla_{\vec k}$ is the gradient in the space of coordinates and wave vectors. It is assumed that for the lower valley, intervalley scattering prevails over intravalley scattering, and for the upper valley, intravalley scattering prevails over intervalley scattering. Then the Boltzmann equation for the lower valley

$$\left(\frac{\partial f}{\partial t}\right)_{inter} + \left(\frac{\partial f}{\partial t}\right)_{intervalley} = 0 \tag{11}$$

For the upper valley

$$\left(\frac{\partial f^b}{\partial t}\right)_{inter} + \left(\frac{\partial f^b}{\partial t}\right) = 0 \tag{12}$$

Davydov [4] showed that in a strong electric field the distribution function has the form:

$$f = f_0 + \frac{\vec p}{p}\vec f_1 \tag{13}$$

f_0 is equilibrium distribution function, $\vec p$ is momentum of charge carriers.

It is clear that one can write

$$f^a = f_0^a + \frac{\vec p}{p}\vec f_1^a,\ f^b = f_0^b + \frac{\vec p}{p}\vec f_1^b \tag{14}$$

The distribution function $f(\vec k, \vec r)$ was found from equation (12) in [4]

$$f_0^a = Be^{-\alpha_a(\varepsilon-\Delta)^2} \tag{15}$$

$$f_1^b = -\frac{em_bl_b}{p}\vec p\frac{\partial f_0^b}{\partial p} \tag{16}$$

Here

$$l_b = \frac{\pi\hbar^4\rho u_0^2}{D^2m_b^2k_0T} \tag{17}$$

$$\alpha_a = \frac{3D^4 m_b^5 k_0 T}{e^2 \pi^2 \hbar^8 \rho^2 u_0^2} \qquad (18)$$

It is clear that for the valley "a" you can write similar formulas (17-18) replacing "a" with "b". l_b is the mean free path, D is the deformation potential, T is the temperature of the solution, ρ is the density of the crystal, u_0 is the speed of sound in the crystal.

Full current

$$\vec{j} = \vec{j}_a + \vec{j}_b \qquad (19)$$

$$\vec{j} = \frac{2e}{(2\pi)^3} \int_0^\infty \frac{\vec{p}}{p} \vec{f} \vec{\vartheta} d\vec{k} \qquad (20)$$

In an external electric and magnetic field, for intravalley scattering f_1^b it has the following form [4]

$$f_1^b = -\frac{em_b l_b}{p} \frac{\partial f_0^b}{\partial p} \cdot \frac{\vec{E} + \left(\frac{el_b}{cp}\right)\left[\vec{E}\vec{H}\right] + \left(\frac{el_b}{cp}\right)^2 \vec{H}\left[\vec{E}\vec{H}\right]}{1 + \left(\frac{el_b}{cp}\right)^2 H^2} \qquad (21)$$

$$\alpha_b = \frac{3D^4 m_b^5 k_0 T \left[1 + \left(\frac{el_b}{cp}\right)^2 H^2\right]}{e^2 \pi^2 \hbar^8 \rho^2 u_0^2 \left[E^2 + \left(\frac{el_b}{cp}\right)^2 \left(\vec{E}\vec{H}\right)^2\right]} \qquad (22)$$

If we replace "b" with "a" in (21-22) f_1^b and α_b are obtained. After a simple calculation of the current density $\vec{j}_a$ and $\vec{j}_b$ from (16) it turns out:

$$\vec{j}_a = \frac{e^2 l_a \alpha_a A}{12\pi^2 \hbar^2 m_a^2} \left\{ \vec{E} \frac{c^2}{e^2 l_a^2 H^2} \left(\frac{4m_a^2}{\alpha_a}\right)^2 + \left[\vec{E}\vec{H}\right] + \frac{c\Gamma\left(7/4\right)}{el_a H^2} \left(\frac{4m_a^2}{\alpha_a}\right)^{7/4} + \vec{H}\left[\vec{E}\vec{H}\right] + \frac{\Gamma\left(3/2\right)}{H^2} \left(\frac{4m_a^2}{\alpha_a}\right)^{3/2} \right\} \qquad (23)$$

After calculating the total current by formula (19)

$$j_z' = \frac{8nc^2 m_a^{1/2}}{3\sqrt{2}\Gamma\left(3/2\right) \cdot l_a} \frac{E_z'}{H^2} \cdot \frac{\alpha_a^{-1/4}}{1 + \gamma_z^{-3/2}\, 3/4\, \beta} \left\{ 1 + t\gamma_z^{-2}\beta + \frac{e^2 l_a^2 \alpha_a^{1/2}}{2c^2 m_a} H^2 \Gamma\left(3/2\right)\left[1 + t\gamma^{-1} z^{1/2} \beta\right] \right\} \qquad (24)$$

Here

$$A = t\gamma^{-1} z^{-1/2} = \frac{m_b}{m_a},\ \gamma = \frac{m_a}{m_b},\ z = \frac{\alpha_a}{\alpha_b},\ t = \frac{l_b}{l_a},\ \beta = z^{-1} e^{-\alpha_a \Delta^2}$$

$$e^{-\alpha_a \Delta^2} = e^{-\left(\frac{E_x}{E}\right)^2} = \left(1 - \frac{E_x}{E}\right)^2,\ E^2 = \frac{3D^4 m_0 m_a^3 k_0 T}{e^2 \pi^2 \hbar^8 \rho^2 u_0^2}$$

Let us write (24) in the following form

$$\vec{j} = \sigma\vec{E} + \sigma_1\left[\vec{E}\vec{h}\right] + \sigma_2 \vec{h}\left[\vec{E}\vec{h}\right] \qquad (25)$$

$\vec{h}$ is unit vector in the magnetic field. For current density j_z' (21), it is directed the electric field and magnetic field H_0 as in (4) as follows

The value E_x is derived from the following condition

$$\frac{dj_z'}{dE_z'} = 0 \qquad (26)$$

For GaAs - E_x^2

$$E_x^2 = 43.84\left(V/sm\right)^2 \qquad (27)$$

For strong electric fields, the condition

$$E >> E_x \qquad (28)$$

quite satisfied. Now we calculate the frequency of current oscillations. An alternating magnetic field H' arises when an alternating electric field E' is excited inside the medium

$$\frac{\partial \vec{H}'}{\partial t} = -crot\vec{E}' \qquad (29)$$

In the presence of electric and magnetic fields, the current density has the form

$$\vec{j} = \sigma\vec{E} + \sigma_1\left[\vec{E}\vec{H}\right] + \sigma_2\vec{H}\left[\vec{E}\vec{H}\right] \qquad (30)$$

By directing the external electric and magnetic field according to (4), taking into account (29), j'_x, j'_y, j'_z are found from (30) ($\vec{h}$ -unit vector in z).

$$j'_x = \sigma\left(1 - \frac{\mu k_z E_0}{\omega}\right)E'_x + \sigma_1\left[\left(1 + \frac{ck_x E_0}{\omega H_0}\right) - \frac{2\sigma_2 ck_z E_0}{\omega H_0}\right]E'_y + \frac{2\sigma_2 ck_y E_0}{\omega H_0}E'_z \qquad (31)$$

$$j'_y = -\sigma_1 E'_x + \left(\sigma - \frac{\sigma_1 ck_z E_0}{\omega H_0}\right)E'_y + \sigma_1\left(1 + \frac{ck_y E_0}{\omega H_0}\right)E'_z \qquad (32)$$

$$j'_z = (\sigma + \sigma_2)E'_z - \frac{2\sigma_2 ck_y E_0}{\omega H_0}\left(E'_x + E'_y\right) \qquad (33)$$

For $j'_x = 0$ and , E'_z and E'_y are found from (31-32), E'_z and E'_y supplying and to (33) under the condition $\mu H_0 >> c$

$$j'_z = \left[\sigma_2 + \frac{2\sigma_2 ck_x E_0}{\omega H_0}\left(1 + \frac{c}{\mu H}\frac{ck_z E_0}{\omega} + \frac{c}{\mu H_0}\frac{ck_y k_z \mu E_0}{\omega_0^2}\cdot\frac{E_0}{H_0} - \frac{ck_y}{\omega}\frac{c}{\mu H_0}\frac{E_0}{H_0}\right) + \right.$$
$$\left. + \frac{2\sigma_2 ck_y}{\omega}\cdot\frac{E_0}{H_0}\left(\frac{ck_y}{\omega} + \frac{c\mu k_z ck_y \mu E_0}{\omega^2}\right)\frac{E_0}{H_0}\right]E'_z \qquad (34)$$

When strong $\mu H_0 >> c$, equating (34) and (24) yields the following dispersion equation

$$(\sigma_2 - \tilde{\sigma}\Phi)\omega^3 + \frac{2\sigma_2 ck_x E_0}{H_0}\left(1 + \frac{E_0}{H_0}\right)\omega^2 + \frac{2\sigma_2 ck_x E_0}{H_0}\omega + \frac{\sigma_2 ck_x E_0}{H_0}ck_y \mu k_z E_0\left(\frac{c}{\mu H_0}\cdot\frac{E_0}{H_0} + 2\frac{E_0}{H_0}\right) = 0 \qquad (35)$$

Denote σ and Φ

$$\tilde{\sigma} = \frac{8nc^2 m_a^{1/2}\alpha_a^{-1/4}}{3\sqrt{2}\Gamma\left(3/2\right)l_a H^2}, \Phi = \frac{1}{1 + \gamma_z^{-3/2} z^{9/4}\beta}\left\{1 + t\gamma^{-2}Z\beta + \frac{e^2 l_a^2 H^2 \alpha_a^{1/2}}{2c^2 m_a}\Gamma\left(3/2\right) + \left(1 + t\gamma^{-1}z^{1/2}\beta\right)\right\} \qquad (36)$$

i.e.

$$H_x = H_0 = \left[\frac{8c^2 m_a^{1/2}\alpha_a^{-1/4}}{3\sqrt{2}\Gamma\left(3/2\right)e\mu l_a}\right] \qquad (37)$$

The following dispersion equation is obtained

$$\left(\frac{\sigma_2'}{\sigma_2}-1\right)\omega^4+2\omega_x\frac{E_0}{H_0}\omega^3+\left(2\omega_x\omega_z\frac{E_0}{H_0}-2\omega_y\omega_x\frac{c}{\mu H}\frac{E_0}{H_0}+2\omega_y^2\frac{E_0}{H_0}\right)\omega^2+$$
$$+\left(2\omega_x\frac{c}{\mu H}\omega_y\omega_z+2\omega_y^2\omega_z\frac{E_0}{H_0}\right)\omega=0 \qquad (38)$$

I case $H_0=H_x$

Then from (38) we obtain the following dispersion equation

$$\Omega_1\omega^2+\Omega_2^2\omega+\Omega_3^2=0 \qquad (39)$$

Solution (39) shows that the growing wave at

$$L_x=\frac{c}{\mu H}L_y \qquad (40)$$

and electric field

$$E_0>\frac{H_0}{4}\frac{\mu H}{c}\cdot\frac{L_x}{L_z} \qquad (41)$$

Ratio $\frac{\omega_0}{\gamma_0}$ (γ is growth increment)

$$\frac{\omega_0}{\gamma}=\left(\frac{L_y}{L_z}\right)^{1/2}\left(\frac{\mu H}{c}\right)^{1/2}\frac{E_0}{H_0}>>1 \qquad (42)$$

i.e. excitation of oscillation by increment

$$\gamma=\frac{\left[2\omega_x\omega_z\omega_x\left(\frac{c}{\mu H}\right)^2+2\omega_x^2\omega_z\frac{E_0}{H_0}\left(\frac{c}{\mu H}\right)^2\right]^{1/2}}{\left(2\omega_x\frac{E_0}{H_0}\right)^{1/2}}=\left(\frac{\omega_z}{\omega_x}\right)^{1/2}\cdot\frac{\frac{c}{\mu H}\left(1+\frac{E_0}{H_0}\right)^{1/2}}{\left(\frac{E_0}{H_0}\right)^{1/2}}\omega_x \qquad (43)$$

Here $\omega_z=ck_z$, $\omega_x=ck_x$

To solve the dispersion equation (38) when (40) is satisfied,

$$x^3+\frac{2E_0}{H_0}x^2+\frac{\omega_zE_0}{\omega_xH_0\varphi}x+\frac{2\omega_z}{\omega_x\varphi}\left(\frac{c}{\mu H}\right)^2=0 \qquad (44)$$

$$\varphi=\frac{\sigma_2'}{\sigma_2}-1$$

The dispersion equation has the following form

$$x^3+ax^2+bx+c=0 \qquad (45)$$

$$x=\frac{\omega}{\omega_x}$$

From (45) the equation is reduced to the form

$$y^3+3py+q=0 \qquad (46)$$

Here $y=\frac{b}{3a}$

Applying the Cardano formula to equation (46) we get 3 roots

$$y_1=u+\vartheta$$
$$y_2=\varepsilon_1u+\varepsilon_2\vartheta=\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)u+\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)\vartheta=-\frac{u+\vartheta}{2}+i\frac{\sqrt{3}}{2}(u-\vartheta)$$
$$y_3=\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)u+\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\vartheta=-\frac{u+\vartheta}{2}+i\frac{\sqrt{3}}{2}(\vartheta-u) \qquad (47)$$
$$u=\sqrt[3]{-q+\sqrt{q^2+p^3}}$$
$$\vartheta=-\sqrt[3]{q+\sqrt{q^2+p^3}}$$

$$3p=\frac{3ac-b^2}{3a^2}$$

$$2q = \frac{2b^3}{27q^3} - \frac{cb}{3a^3} + \frac{d}{a}$$

Substituting the values of q and p into equations (47)

$$E_0 = (6)^{1/2} \frac{c}{\mu} \text{ at } a = -1 \text{, i.e } H_0 >> H_x \qquad (48)$$

A growing wave with a certain frequency is excited. In case when $H << H_x$

$$x = -\frac{bH^2}{3H_x^2} + i\frac{\sqrt{3}}{2}\left(\frac{2bH^2}{3H_x^2} + \frac{2bH^2}{3H_x^2}\right) = -\frac{bH^2}{3H_x^2} + i2\sqrt{3}\frac{bH^2}{3H_x^2} \qquad (49)$$

From (49) it can be seen that the ratio of the increment to the frequency

$$\omega_0 = -ck_x \frac{bH^2}{3H_x^2}, \quad \omega_1 = -\frac{2}{\sqrt{3}}\frac{bH^2}{H_x^2}ck_x$$

$$\frac{\omega_1}{\omega_0} = \frac{2}{\sqrt{3}} = 2\sqrt{3} > 1 \qquad (50)$$

This inequality occurs when

$$E_0 > H_0\left(\frac{L_x}{24L_z}\right)^{1/3}\left(\frac{H_x}{H_0}\right)^{4/3}\left(\frac{c}{\mu H}\right)^{2/3} \qquad (51)$$

Substituting (40) into (51) we obtain the following relationship between L_x and L_z

$$L_y < L_z 3^{5/6} \cdot 2^{1/2}\left(\frac{H_x}{H}\right)^{2/3}\left(\frac{c}{\mu H}\right)^{1/3} \qquad (52)$$

Thus, a growing wave of an electromagnetic nature is excited in two-valley semiconductors of the GaAs type. The growth rate of this wave changes significantly with the change in the ratio $\frac{H}{H_x}$. The excited wave depends very strongly on the size of the crystal (L_x, L_y, L_z).

DISCUSSION OF THE RESULTS

It is shown that the radiation of electromagnetic waves in the form of these media occurs in three cases I, II, III. Thus, an unstable wave is not excited at all values of the magnetic field, only at certain values relative to the characteristic magnetic field H_x.

[1] *B.K. Ridley, T.B. Watkins.* The Possibility of Negative Resistance Effects in Semiconductors, Pros.Phys.Soc., 1961, v. 78, p. 293.

[2] *C.H. Hilsum.* Transferred Electron Amplifiers and Oscillators. Proc. IRE. 1962, V.50, p.185.

[3] *A.I. Anselm.* Introduction to the theory of semiconductors. Moscow. M. 1962.

[4] *B.I. Davydov*. On the theory of the motion of electrons in gases and in semiconductors. ZhETF. 1937, T.7., 1069.

[5] *E.R. Hasanov, R.A. Hasanova.* External and Internal Instability in the Medium Having Electron Typ Conductivity, IOSR Journal of Applied Physics, (May-June. 2018), Volume 10, Issue 3, Ver. II, p.18-26.

[6] *E. Conwell.* Kinetic properties of semiconductors in strong electric fields. (“Mir” Moscow, 1970), p.339-344.

[7] *L.E. Gurevich, E.R. Hasanov.* Spontaneous current oscillations in semiconductors with deep traps in strong electric and magnetic fields, *Solid State Physics*, 1969, vol. 11 № 12, p.3544-3548.

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