LOSSES FOR PARAMETRIC INTERACTION IN MEDIUM WITH NEGATIVE REFRACTION

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An analytical expression was acquired for determining the optimal value of the pump intensity during three-wave interaction in medium with negative refraction under the condition of phase synchronism. The losses for parametric interaction in the metamaterial can be compensated for the backward wave. The optimal value for pumping intensity is obtained by analytical solution. The analysis showed that it depends on the intensity of interacting signal and idler waves.

Keywords: metamaterial, parametric interaction, negative refraction.
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INTRODUCTION

The discovery of metamaterials contributed to the emergence of the possibility of controlling light radiation by changing the optical properties of such artificial structures. Resonant interactions, the existence of backward waves, negative refraction in chronological order were considered in the works of G. Lamb [1], L.I. Mandelstam [2], D.V. Sivukhin [3], V.N. Agranovich and V.L. Ginzburg [4], V.G. Veselago [5] and others. [6].

One of the features of these materials is the possibility within the same structure at the same time, but for different frequency ranges, to ensure the existence of both positive and negative values of the real part of the refractive index of a nonlinear medium (i.e., simultaneously negative values of the dielectric (ε dielectric permittivity) and magnetic permeability (μ magnetic permeability). Hence, the unconventional result of the interaction of an electromagnetic pump wave with a metamaterial manifests itself in the nonlinear interaction of optical waves, for example, in the generation of harmonics [7,8], as well as in the generation of sum and difference frequencies. Then, if the pump wave is in the frequency range with a negative refractive index, and the harmonic wave is in the frequency range with the opposite sign of the refractive index, then the fundamental wave transfers energy in the opposite direction to its phase velocity. With such a geometry, the pump wave is known to be a backward wave. In this case, the maximum intensity of the harmonic is achieved not at the output, but at the input to the nonlinear medium. This medium plays the role of a nonlinear mirror. A similar conduction of a nonlinear medium is manifested in the case of a degenerate four-wave interaction when observing the effect of wavefront reversal of laser radiation [9-11].

To date, thanks to the improvement of the technology of manufacturing metamaterials, their development from the radio range is moving towards the visible range. In [12] they report on the results of developments already in the near IR and in the visible ranges of the spectrum.

In the constant field approximation a theoretical study of the nonlinear optical interaction in such artificial structures was carried out in a number of works, of which we note [9–15, 16-23]. In the constant field approximation [15, 24, 31], we studied the generation of the second and third harmonics, the effects of self-action in a metamaterial [25, 26].

In classical electrodynamics, according to the dispersion relations of Kramers - Kronig, which determine the behavior of the optical constant of the medium - the real (refractive index) and imaginary (absorption coefficient) parts of the electrical permeability of frequency, at the resonant frequency, the absorption coefficient increases sharply, which leads to significant energy losses of the electromagnetic wave. Thus, there are inevitable losses in metamaterials that weaken the electromagnetic wave.

The main problem in the study of metamaterials is high losses. [7-10]. Various constructive variants of metamaterials are being investigated, where it is possible to attenuate signal losses. One of the ways to overcome the losses is considered and proposed as a result of the analysis in the constant intensity approximation [12] in the work [13,14]. The influence of losses in metamaterials for the case of four-wave interaction is studied for the process of amplification and generation of the reverse signal wave in the constant intensity approximation.

The aim of this scientific work is to study of phase effects in three-wave parametric interaction in metamaterials, and also to compute the optimal study of the pump intensity in a three-wave parametric interaction in a metamaterial. The nonlinear optical interaction in metamaterials has been studied in the constant intensity approximation [8,9]. In the constant intensity approximation the generation of the second and third harmonics, self-action effects and parametric interaction in metamaterials [27-30], four-wave interaction in metamaterials [24, 29] were studied.

THEORY

Consideration is carried out for the case of parametric three-wave interaction in a metamaterial at a frequency ω_p. We believe that the properties of the metamaterial are manifested for the wave at the pump

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frequency ω_p ($\omega_p = \omega_i - \omega_s$). We assume that the waves at the frequencies of the signal and pump waves run along the positive z axis. In the constant intensity approximation, parametric interaction in a metamaterial without taking into account losses, but at different phase distances and initial values of the intensities of the interacting waves at high-frequency and low-frequency pumping, was carried out by us in [27, 28].

The usual shortened equations for three interacting waves in a metamaterial take the form [27]:

$$\frac{dA_i}{dz} + \delta_i A_i = -i\gamma_i A_s A_p e^{-i\Delta z}$$
$$\frac{dA_s}{dz} - \delta_s A_s = -i\gamma_s A_i A_p^* e^{i\Delta z} \quad (1)$$
$$\frac{dA_p}{dz} + \delta_p A_p = -i\gamma_p A_i A_s^* e^{i\Delta z}$$

Here, $A_{i,s,p}$ the corresponding complex amplitudes of the interacting waves, δ_j - are the absorption coefficients of the medium at frequencies ω_j (j=1,3),

$$\gamma_i = \frac{8\pi\chi_{eff}^{(2)}\omega_i^2\varepsilon_i}{k_i c^2}, \gamma_s = \frac{8\pi\chi_{eff}^{(2)}\omega_s^2\varepsilon_s}{k_s c^2}, \gamma_p = \frac{8\pi\chi_{eff}^{(2)}\omega_p^2\varepsilon_p}{k_p c^2}$$

- coefficients of nonlinear connection of waves at the corresponding frequencies, $\Delta = k_i - k_s - k_p$ - phase mismatch between interacting waves, $\chi_{eff}^{(2)}$- effective quadratic susceptibility of the medium, $z = 0$ corresponds to the entrance to the left of the metamaterial.

When constructing the second equation of system (1), we took into account that the medium has a negative refractive index at the frequency ω_s, so a minus sign appears in front of the loss factor in the second equation.

The boundary conditions in this case are:

$$A_{i,p}(z=0) = A_{i0,p0}\,,\ A_s(z=l) = A_{sl}\,. \quad (2)$$

$A_{i0,p0}$ - the input values of the waves at the sum frequency and at the frequency of the pump wave at $z = 0$, A_{sl} the initial amplitude of the wave at the difference frequency ω_s at the entrance to the metamaterial on the right.

We apply the constant intensity approximation of the fundamental radiation, $I_{i,p}(z) = I_{i,p}(z=0) = const$ and for the intensity of the wave at the difference frequency ω_s at the output on the left at z=0 we obtain [30]:

$$I_s(z=0) = I_{sl}\frac{\exp\left[(\delta_i + \delta_p - \delta_s)l\right] + \frac{\gamma_s^2 I_{i0} I_{p0}^*}{I_{sl}}\left(\frac{\text{sh}\lambda l}{\lambda}\right)^2}{\left[\text{ch}\lambda l + \frac{(\delta_i + \delta_s + \delta_p)}{2}\frac{\text{sh}\lambda l}{\lambda}\right]^2} \quad (3)$$

here,

$$\Gamma_p^2 = \gamma_i\gamma_s I_{p0}, \Gamma_i^2 = \gamma_s\gamma_p I_{i0}, I_{j0} = A_{j0}\cdot A_{j0}^*, j = 1 \div 3, \lambda = \sqrt{\Gamma_p^2 - \Gamma_i^2 + \frac{(\delta_i+\delta_s+\delta_p)^2}{4}}.$$

In order to study the optimal generation mode with respect to the intensity of a strong pump wave, we take the derivative of the output signal intensity on I_{p0}, i.e.,

$$\frac{dI_s}{dI_{p0}} = \frac{\left(\frac{\gamma_s^s I_{i0} I_{p0}^*}{I_{sl}}\right)'\left[\left(\frac{\text{sh}\lambda l}{\lambda}\right)^2\right]'}{\left[\left(\text{ch}\lambda l + \delta_s\frac{\text{sh}\lambda l}{\lambda}\right)^2\right]'}$$

As a result of mathematical transformations, we obtain

$$a\cdot\left[\left(\frac{\text{sh}\lambda l}{\lambda}\right) + sI_{p0}^*\cdot\lambda' b\right]\frac{\text{sh}\lambda l}{\lambda}\cdot c = 2\left[1 + aI_{p0}^*\left(\frac{\text{sh}\lambda l}{\lambda}\right)^2\right](\lambda' l\cdot\text{sh}\lambda l + \delta_s\lambda' b)$$
$$a\cdot c\cdot\frac{\text{sh}\lambda l}{\lambda} = \left[\left(\frac{\text{sh}\lambda l}{\lambda}\right) + 2I_{p0}^*\cdot\lambda' b\right]s\lambda'\left[1 + aI_{p0}^*\left(\frac{\text{sh}\lambda l}{\lambda}\right)^2\right](l\cdot\text{sh}\lambda l + \delta_s b) \quad (4)$$

Where made are the following substitutions:

$$a=\frac{\gamma_s^2 I_{i0}}{I_{sl}},b=\frac{\text{ch}\lambda l\cdot\lambda l-\text{sh}\lambda l}{\lambda^2},c=\text{ch}\lambda l+\delta_s\frac{\text{sh}\lambda l}{\lambda},\lambda'=\sqrt{\Gamma_p^2-\Gamma_i^2+\frac{(2\delta_s-i\Delta)^2}{4}}.$$

Here, to simplify the calculations, we assume that the real and imaginary input intensity values I_{p0} are equal, i.e. $I_{p0}=I_{p0}^*$. This does not limit the scope of the problem and does not significantly change the interaction analysis. By solving the right and left parts of the obtained analytical expression separately, it will be possible to numerically determine the optimal value of the intensity I_{p0} for each set of problem parameters (phase mismatch, wave losses and pump wave intensity (via the parameter Γ_p^2) and intensity of the direct wave (via the parameter Γ_i^2).
This calculation can be required in the development of parametric transducers based on metamaterials.

CONCLUSION

Thus, the paper considers, taking into account phase effects, the parametric interaction of waves in a quadratic medium, which is "left" for a signal wave. The features of the process in this case are analyzed. We performed an analytical calculation for the optimal value of the pump intensity for three-wave interaction in medium with negative refraction under the condition of phase matching. It is shown that the optimal value of the pump intensity depends on the intensities of the interacting signal and idler waves. This fact is not found in the constant intensity approximation. By analytically solving the obtained expression, one can calculate the optimal value of the pump wave intensity for each set of problem parameters (phase mismatch, wave loss, and pump wave intensity (via the parameter Γ_p^2) and direct wave intensity (via the parameter Γ_i^2).

This calculation can be required in the development of parametric transducers based on metamaterials.

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