OPTICAL PROPERTIES OF SEMICONDUCTOR SUPERLATTICES (REVIEW)

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The optical properties of semiconductor superlattices are considered - solid-state structures in which, in addition to the periodic potential of the crystal lattice, there is an additional one-dimensional potential, the period of which significantly exceeds the lattice constant.

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The progress of solid-state electronics is driven by research in the field of creating materials with new, sometimes unusual, properties. One such material is a superlattice (SL). Semiconductor superlattices are nanostructures consisting of alternating layers of two distinct semiconductor materials, such as GaAs and (AlGa)As, which often have extremely comparable lattice constants [1]. Interest in SL is primarily due to the fact that electronic energy. The spectrum of SL can be controlled to a certain extent at the stage of its creation. Presence of superlattice potential significantly changes the energy spectrum, due to which superlattices have a number of interesting properties that are absent in conventional semiconductors. The parameters of the superlattice potential can be easily changed over a wide range, which in turn leads to significant changes in the energy spectrum. Thus, it is easy to change the band structure of semiconductor superlattices. The parameters of the superlattice potential can be easily changed over a wide range, which in turn leads to significant changes in the energy spectrum. Thus, it is easy to change the band structure of semiconductor superlattices.

Explosive growth of both theoretical and experimental interest to superlattices is associated with the latest advances in technology based on molecular beam epitaxy in ultra-high vacuum, also reviews as well as metal-organic epitaxy from the gas phase and other methods.

Due to a number of quantum mechanical phenomena, such as resonant tunneling, the development of Wannier-Stark energy level ladders, and the occurrence of Bloch oscillations [1, 2], whose frequency is proportional to the spatial period, d, of the SL as well as to F, the current-voltage characteristics of semiconductor superlattices are typically highly nonlinear. Bloch oscillations do not occur in natural crystals because *d* is so small (~ 0.3 nm), meaning that the Bloch frequency is substantially smaller than the electron scattering rate. However, in semiconductor superlattices, *d* and the associated Bloch frequency can be large enough for the electrons to oscillate in a Bloch fashion, which is then important for the high-frequency charge transport process as well as the dc one. As the electric field increases, the drift (average) velocity of the electrons decreases due to the localization caused by the commencement of Bloch oscillations.

However, difficulties in experimental obtaining high-quality SL samples led to the fact that the theoretical study of SL [3] began long ago before the appearance of the first experimental samples of satisfactory quality, obtained on the basis of $GaAs/Al_xGa_{1-x}As$ by epitaxy from molecular bunches. In particular, in [3,4] it was proposed SL energy spectrum in the one-miniband approximation

$$\varepsilon(\vec{p}) = \frac{p_y^2+p_z^2}{2m} + \Delta\{1-\cos(\mathrm{p_x d}/\hbar)\} \qquad (1)$$

where m is the effective mass of the electron in the ZOY plane, A is the half-width of the conduction miniband, *d* - period SR. In this case, it is assumed that CP is periodic along the OX axis, P_x, P_y, P_z are the components of the quasi-momentum electron on the coordinate axis.

Figure 1 shows the superlattice structure and band structure. We can see that GaAs and AlAs will form a potential well, as shown in figure 1(b), this is because of the different band gap in these two materials. Furthermore, the motion of the charged carrier in superlattice quantum wells will be effected by potential barriers on both side of the potential well, so that it will be hindered We consider the transport of an electron gas in a semiconductor superlattice with a periodical potential well *U(z)* along the *z* direction under the influence of a constant magnetic field *B=B//z*. The one-particle Hamiltonian describing an electron in a semiconductor superlattice is then given by

$$\mathrm{H} = \frac{1}{2\mathrm{m}^*}(\mathrm{p}+\mathrm{eA})^2 + \mathrm{U(z)} + \mathrm{g}^*\mu_\mathrm{B}\sigma\cdot\mathrm{B} \qquad (2)$$

The electron wave function and the eigenvalues, *E*, of Eqs. (3), (4) and (5) in the conduction band are expressed as

$$\langle x, y.z|\lambda\rangle = \psi_{n,}k_{x,}k_{z,\sigma}(x,y,z) = \frac{1}{\sqrt{L_x}} exp[ik_x x]u_{n,k_x}(y)\xi_z\,(z)\chi_\sigma \qquad (3)$$

with

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$$u_{n,k_x}(y) = \left(\frac{1}{2^n n!\sqrt{\pi} l_B}\right)^{1/2} \times exp\left[-\frac{(y-y_0)^2}{2l_B^2}\right] H_n\left(\frac{y-y_0}{l_B}\right) \tag{4}$$

$$E_\lambda = E_{n,\sigma}(k_x, k_y) = \left(n+\frac{1}{2}\right)\hbar\omega_c + \varepsilon_{SL}(k_z) + \sigma g^* \mu_B B \tag{5}$$

The conduction electrons within the semiconductor superlattices layers may experience high-frequency collective oscillations due to this negative differential velocity [3], which makes semiconductor superlattices appealing for the production and detection of electromagnetic radiation in the GHz to THz frequency range [5, 6, 7]. Negative differential velocity for electrons in superluminate have also given rise to essentially new charge transport regimes including deterministic chaos, which are typified by intricately erratic electron dynamics. [1, 8, 9]. It offers a sensitive new method of regulating electron transport and defies the Kolmogorov-Arnold-Moser (KAM) theorem. Surprisingly, the effective classical Hamiltonian for the electron motion in a semiconductor superlattice depends explicitly on the miniband's energy versus wavevector dispersion relation, indicating that it has an underlying quantum-mechanical foundation [13,14].

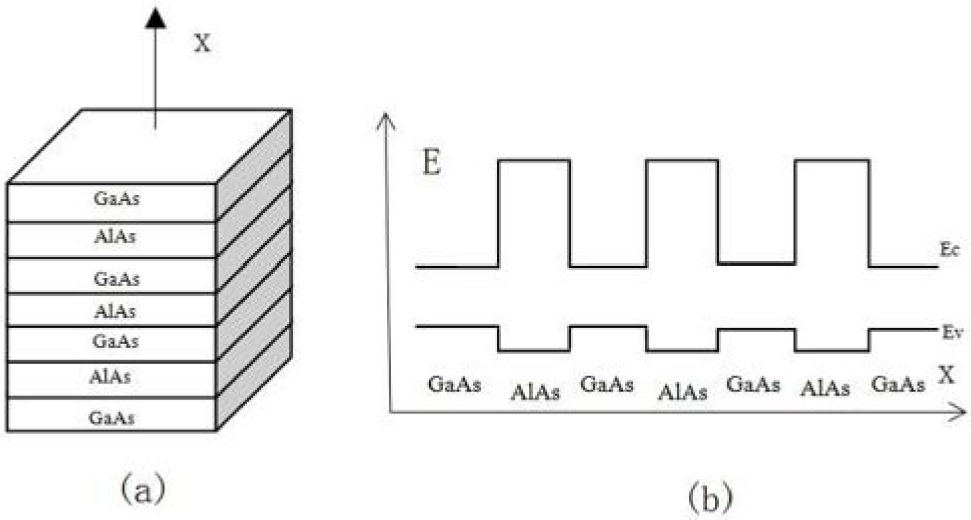

Fig. 1. (a) Schematic of the superlattice structure. (b) Schematic of the band structure of the conduction band the valence band.

The stochastic electron motion turns on abruptly when the field parameters meet specific resonance criteria, which are explained in more detail below, according to an analysis of Hamilton's equations. By leaving an intricate web of conduction channels in phase space—also referred to as a "stochastic web"—the beginning of chaos delocalizes the electrons and results in a significant resonant amplification of the electron velocity and current flow as observed in experiment [14]. This phase-space patterning works even at room temperature [14], and it offers a radically novel idea for regulating electrical conductance [20]. The resonant delocalization of the conduction electrons would result in a sequence of delta-function peaks in the I(V) curves in the absence of dissipation. However, in actual semiconductor superlattices, the resonances are widened because the electrons scatter inelastically via the tunnel walls and quantum well interface roughness as well as elastically [1], primarily because of ionized donor atoms. To get I(V) curves that correspond with experiment, the influence of scattering on the electron dynamics must be considered while computing the transport characteristics of the semiconductor superlattices.

In work [21] investigated how dissipation affects electron transport in a semiconductor superlattice under the influence of a magnetic field that is oriented perpendicular to the superlattice axis and an applied bias voltage. In this work demonstrated that the applied fields, despite being stationary, behave as a THz plane wave, closely coupling electron Bloch and cyclotron motion inside the lowest miniband. This leads to a special kind of Hamiltonian chaos in the electrons, which in turn forms a complex network of conduction channels (a stochastic web) in phase space. At critical values of the applied voltage, this produces a significant resonant increase in the current flow. This patterning in phase-space offers a delicate way to regulate electrical resistance. In this work, introduced a linear damping term into the semiclassical equations of motion to examine the impact of dissipation on the electron dynamics. It is shown that deterministic chaos contributes significantly to the electron transport process even when dissipation is present. Complex multistability phenomena arises in the system when the cyclotron and Bloch frequencies are comparable. Specifically, multiple separate stable regimes can coexist for fixed values of the control parameters, each of which corresponds to a different initial condition. In this work is demonstrated that the electron transport properties clearly exhibit the fingerprints of this multistability, as observed experimentally.

Semiconductor materials have been widely used in electronics in recent years. The phenomena of contact p-n and the doped capacity to change the physical characteristics of crystals are the fundamental driving forces behind the development of semiconductor electronics. As a result, many scientists are now interested in the magneto-thermoelectric effects in low-dimensional systems. In [22] investigates the impact of Ettingshausen in semiconductors when laser radiation is present. The findings demonstrate that the degree of these "photostimulated" effects can be greater than the similar effects when radiation is not present. The EC in bulk semiconductors is expressed analytically in [22], which also demonstrates how the Ettingshausen coefficient depends on lattice heat conductivity, radiation frequency, and the amplitude of the laser radiation field conductivity.

In [23], the Boltzmann kinetic equation was utilized to investigate the Ettingshausen effect in compositional semiconductor superlattices caused by an electric field. The outcome demonstrates how the Ettingshausen effect is affected by magnetic and electric fields, as well as how it may be possible to increase (transverse) thermoelectric effectivity at the expense of electric fields.

In work [24] is investigated magneto-thermoelectric effects in the doped semiconductor superlattice under the impact of electromagnetic waves using a quantum kinetic equation for electrons. We have also determined analytical expressions of the Ettingshausen coefficient in doped semiconductor superlattice for the electron-acoustic phonon interaction. Compared to the expressions found in the case of bulk semiconductors, these expressions are significantly different. They observed that the Ettingshausen coefficient relies on the temperature, the characteristic parameters of doped semiconductor superlattice, and the characteristic parameters of electromagnetic waves. The results are computed numerically for the GaAs:Be/GaAs:Si doped semiconductor superlattice. The findings align with recent experimental findings; nonetheless, the Ettingshausen coefficient differs from that of bulk semiconductors or bismuth.

Using canonical ensemble statistics for under a tilted magnetic field, investigated the orbital magnetization and heat capacity of an electron system analytically and quantitatively [25]. For a bulk semiconductor superlattice, the orbital magnetization and heat capacity are obtained as functions of the magnetic field intensity at finite temperature. In addition, the calculations are made for the orbital magnetization, heat capacity, temperature, and tilt angle of the slanted magnetic field.

The Hall effect in doped semiconductor superlattices (DSSL) under the impact of restricted LO-phonons and laser radiation has been theoretically investigated based on the quantum kinetic equation technique [26]. The Hall coefficient, magnetoresistance, and Hall conductivity tensor of a GaAs:Si/GaAs are expressed analytically as follows: In terms of the doping concentration, lattice period and external fields, DSSL is achieved. The effect of electron and LO-phonon confinement was characterized by varying the quantum numbers N, n, and m. When compared to the case of bulk phonons, numerical evaluations demonstrated that LO-phonon confinement increased the probability of electron scattering, increasing the number of resonance peaks in the Hall conductivity tensor and decreasing the magnitude of both the magnetoresistance and the Hall coefficient, in contrast to the situation of bulk phonons. It was discovered that the magnetoresistance's nearly linear temperature increase matched the experiment fairly well. Figure 2 shows the dependence of the conductivity tensor s_{xx} on the cyclotron energy for confined phonon and bulk phonon.

It is commonly recognized that, in low-dimensional semiconductor systems, the effect of phonon confinement alters the probability of carrier scattering, resulting in unique material behaviors relative to the case of unconfined phonons [27, 28]. As a result, numerous published works have explored the impact of confined phonons on the optical, electrical, and magnetic characteristics of low-dimensional semiconductor systems. These include the impact of confined phonons on carrier capture processes [30], the absorption coefficient of strong electromagnetic waves [29], and the resonant quasi-confined optical phonons in semiconductor superlattices [31]. The branches of optical phonons in semiconductor systems are limited when they do not overlap; the confined optical phonon's wave vector contains both the quantized and in-plane components [27, 32]. The guided mode model, the slab mode model, and the Huang-Zhu model are examples of diverse confined phonon models that result from the various boundary conditions applied to the electrostatic potential or vibrational amplitude of the phonons [33]. We have examined the Hall effect in doped semiconductor superlattices (DSSL) with bulk phonons in the earlier study [8]. It has been demonstrated via the studies of [28, 29,32] that phonon confinement plays a significant role in the characteristics of low-dimensional semiconductor systems and shouldn't be disregarded. Using the quantum kinetic equation technique, the analytical formulations of the Hall conductivity tensor, the magnetoresistance, and the Hall coefficient in DSSLs under the effect of confined LO-phonons are found [29, 34].

For nonpolar optical phonons, in [35] derived the longitudinal magnetoconductivity of superlattices. The plateau condition that appears in superlattices is obtained analytically from the relaxation rates which are intimately associated with the magnetophonon resonance. The temperature and miniband width are taken into consideration while examining the qualitative aspects of the magnetophonon resonance, magnetophonon resonance effects. Specifically, the magnetophonon resonance line form behaviors are explored in detail, including the emergence of the plateau between nearby magnetophonon resonance peaks, the removal of magnetophonon resonance peaks, and variations in the magnetophonon resonance amplitude. Their findings align well with current experimental and theoretical findings.

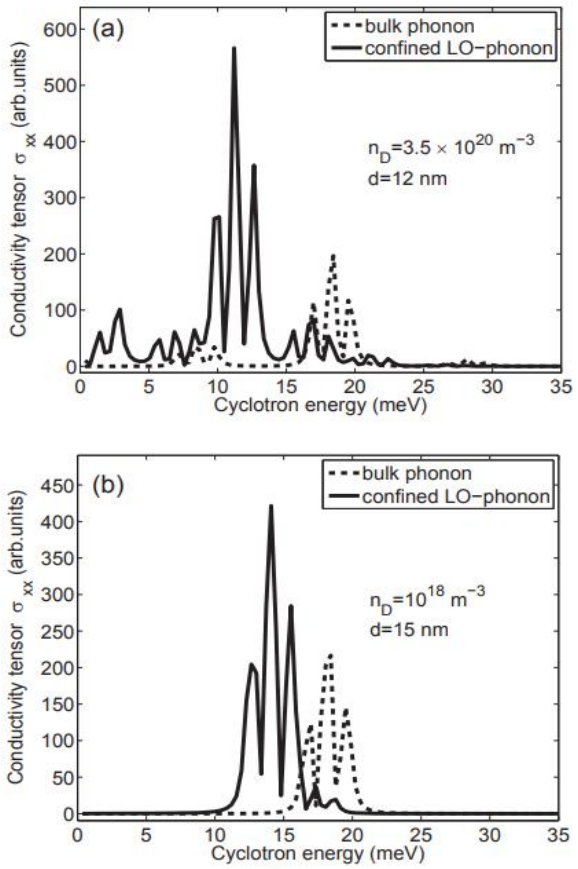

Fig. 2. The dependence of the conductivity tensor s_{xx} on the cyclotron energy for confined phonon (solid curve) and bulk phonon (dashed curve), here *d* =12 nm (fig. 1a) and *d* =15 nm (fig. 1b) [26].

There have been numerous reports of magnetophonon resonance effects investigations in these kinds of systems [36,37.38-41]. Noguchi et al. [39] regarding the magnetophonon resonance in short-period semiconductor superlattices. Their findings for the longitudinal magnetoresistance in GaAs/Al$_x$Ga$_{1-x}$ As superlattices under strong magnetic fields parallel to the electronic fields and normal to the interfaces were reported. Their findings showed that in short-period superlattices, unusual behaviors of the magnetophonon resonance line shape were observed, including the emergence of a plateau between neighboring magnetophonon resonance peaks and the removal of magnetophonon resonance peaks. The energy dispersion relation and the plateau condition derived from the density of states provided an explanation for them. A thorough theoretical analysis of the miniband transport of electrons in a GaAs-based superlattice under the effect of a quantizing magnetic field normal to the layer plane was recently reported by Shu and Lei [40]. More recently, Gassot et al. [41] reported the temperature, pressure, miniband, and electric field dependences of magnetophonon resonance in superlattices and observed strong oscillations of magnetophonon resonance in the background of the longitudinal magnetoconductance in short-period GaAs/AlAs superlattices resulting from electron interactions with both GaAs and AlAs longitudinal optical (LO) phonons. The goal of this work is to derive the plateau for the nonlinear dc conductivity using the linear response limit of a generic equation previously developed, to present a systematic theoretical analysis of miniband electron transport in such a system, to obtain analytically the plateau condition for a GaAs-based superlattice under the influence of a quantizing magnetic field normal to the layer plane and to compare with the theoretical and experimental results reported by some authors [39–41].

In bulk semiconductors, the magnetophonon resonance (MPR) phenomenon has drawn a lot of interest from both practical and theoretical perspectives since it was first predicted by Gurevich and Firsov [42]. There have been numerous reports of MPR impact research in such low-dimensional electronic devices [43–51]. On the other hand, short-period superlattices have received less attention. Noguchi et al. [52] carried out research on MPR in semiconductor superlattices in 1992. Their findings of longitudinal magnetoresistance in GaAs/Al$_x$Ga$_{1-x}$ As superlattices under strong magnetic fields parallel to the electric fields and normal to the interfaces have been presented. Shu and Lei [53] recently provided a systematic theoretical analysis of miniband transport of electrons in a GaAs-based superlattice under the influence of a quantized magnetic field normal to the layer plane. They also observed strong oscillations of MPR on the background of the longitudinal magnetoconductance in short-period GaAs/AlAs superlattices, which are caused by electron interactions with both GaAs and AlAs LO phonons. In work [50], find that as the applied magnetic fields' tilt angle increases, the MPR peak positions of the GaAs and AlAs LO phonon modes shift to regions of higher magnetic strength, while the second derivative of the magnetoconductivities—which includes a miniband width and a potential periodicity—declines and disappears under the same experimental conditions [41]. We contrast our findings with the theoretical and experimental findings reported by a few writers [52-54]. Since its initial prediction by Gurevich and Firsov [42], the magnetophonon resonance (MPR) phenomena in bulk semiconductors has attracted considerable attention from both theoretical and practical standpoints.

The angular dependence of magnetophonon resonance (MPR) peaks for longitudinal optical (LO) phonons in a tilted magnetic field applied to the superlattice axis is obtained from the longitudinal magnetoconductivities of GaAs/AlAs semiconductor superlattices. Using the second derivative of the magnetoconductivities, which includes a miniband width and a potential periodicity, the MPR peaks of the LO phonon modes in both GaAs and AlAs are displayed. The MPR peak placements shift to regions of higher magnetic strength as the tilt angle of the applied magnetic fields increases, but the peak amplitudes decrease and eventually vanish under the same conditions in the tests. Our findings qualitatively match the experimental findings quite well. As a function of the tilt angles, we also obtain the resonance fields for the MPR peak numbers of the GaAs and AlAs LO phonon modes. In order to explain the emergence of a plateau zone between adjacent peak amplitudes, we also derive the dispersion relations and the density of states (DOS), which include a miniband width, a periodicity of the potential, and tilt angles of the slanted magnetic fields. The plateau area can be elucidated by phenomena like the second derivative of the magnetoconductivity of GaAs/AlAs semiconductor superlattices for tilt angles of the applied magnetic fields and the shifts, decreases, and disappearances of the peak amplitudes of the magnetoconductivities [55]. Fig. 3 and fig.4 shows angular dependence of the frequency-dependent magnetoconductivity ¯ $\sigma_{zz}(\omega)$ for LO phonon scattering of a Ge-based semiconductor superlattice.

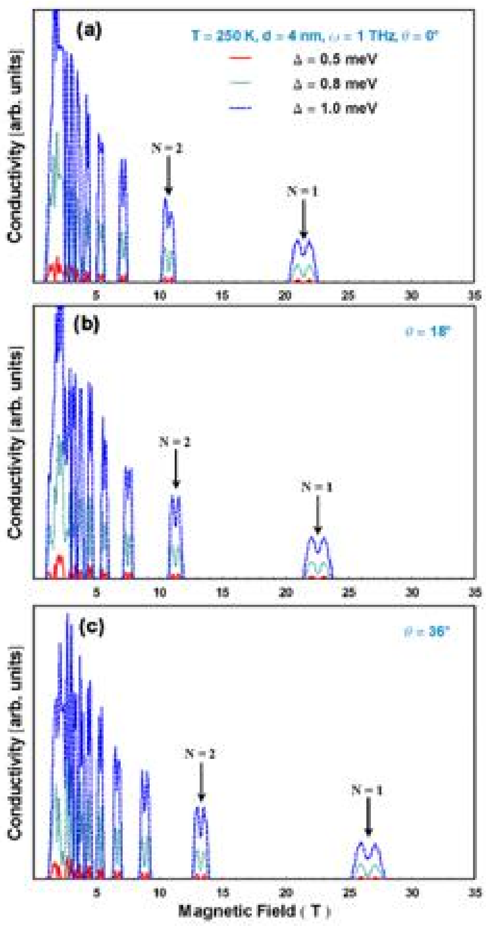

Fig. 3. Angular dependence of the frequency-dependent magnetoconductivity $\sigma_{zz}(\omega)$ for LO phonon scattering of a Ge-based semiconductor superlattice with a photon frequency of ω = 1 THz as a function of the magnetic field strength at T = 250 K. The solid, dotted, and dashed lines are for Δ = 0.5 meV, 0.8 meV, and 1.0 meV, respectively [55].

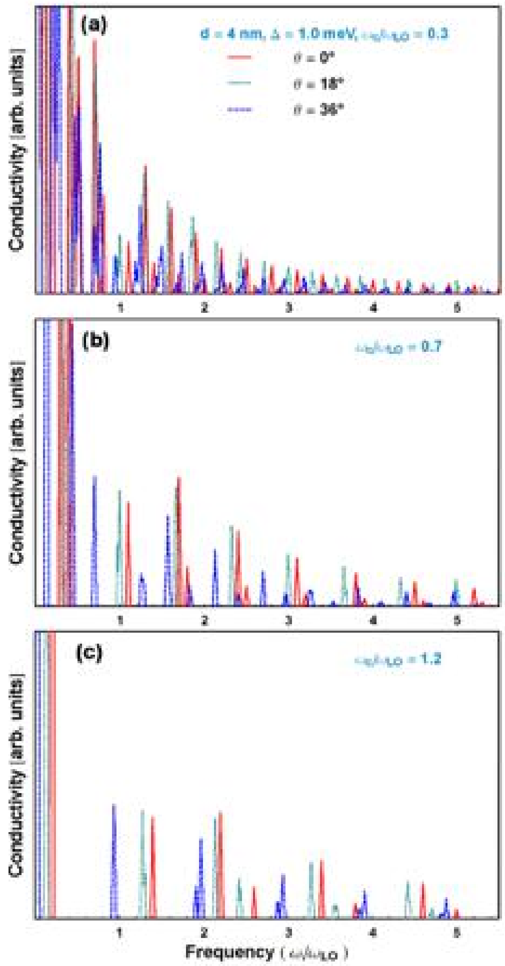

Fig. 4. Angular dependence of the frequency-dependent magnetoconductivity $\bar{h}\sigma_{zz}(\omega)$ for LO phonon scattering of a Ge-based semiconductor superlattice with a miniband width Δ=1.0 meV at T = 250 K. The solid, dotted, and dashed lines are for θ = 0°, 18°, and 36°, respectively [55].

Based on both Wannier-Stark quantum transport theory and the Landau levels obtained when using magnetic and steady strong electric fields parallel to the axis of the semiconductor superlattice (see fig.5), respectively, we were able to derive the conditions for the Wannier-Stark magnetophonon resonance from the electric conductivity and a relaxation function, which are functions of the temperature, the period of the superlattice, the miniband width, the electron density, and the magnetic and electric fields. Numerical research was done on the behaviors of the line shapes derived from the Wannier-Stark magnetophonon resonance of the relaxation function and the electric conductivity as functions of the phonon energy, electric field, and magnetic field. With increasing electron transitions between the Wannier-Stark and Landau levels, the number of resonance peaks was reduced [56]. Fig. 6 shows electric conductivity σ_{zz} as a function of the electric field for various periods and miniband widths in semiconductor superlattices

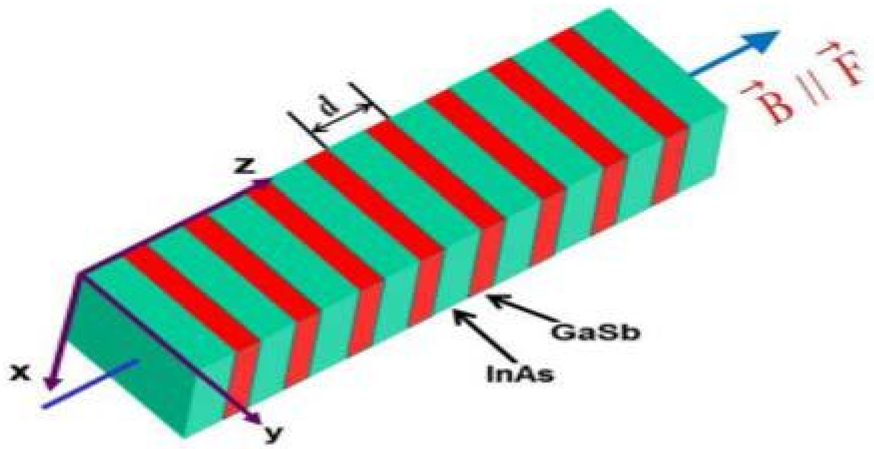

Fig. 5. Schematic illustration of an InAs/GaSb semiconductor superlattice [56].

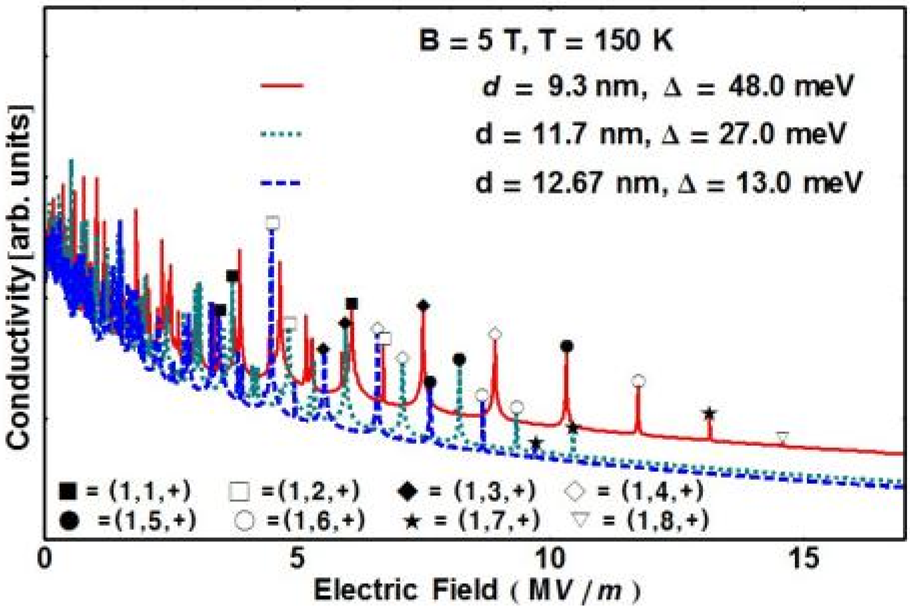

Fig. 6. Electric conductivity σ_{zz} as a function of the electric field for various periods and miniband widths in semiconductor superlattices at B = 5 T and T = 150 K. The solid, dotted, and dashed lines are for d = 9.3 nm (Δ = 48.0 meV), d = 11.7 nm (Δ = 27.0 meV) and d = 12.6 nm (Δ = 13.0 meV), respectively [56].

An investigation is conducted on hopping conduction between localized Stark states in superlattices under longitudinal electric and magnetic fields. Consideration is given to electron scattering with polar-optic phonons, short-range impurities, and acoustic phonons [57]. The expression for hopping current formally exhibits a series of maxima at the points $p\omega_c = n\theta$ [Stark-cyclotron resonance (SCR)] or at $p\omega_c = n\theta \pm\omega_{0l}$ [Stark-cyclotron-phonon resonance (SCPR)]

(ω_c and θ are cyclotron and Bloch frequencies, respectively, ω_{0l} vol are optic-phonon frequency modes in superlattices, and p and n are integer numbers). However, because of finite collisional widening, the SCRs are substantially suppressed and difficult to detect experimentally. Numerical simulations conducted inside the dielectric contininium model reveal that the amplitude of SCPR is far smaller than that of SCR and does not contribute to the hopping current. These findings are qualitatively consistent with recent research.

Bloch states are used to realize conduction along the field, and the momentum representation (miniband transport regime) is the basis for the description of transport in several formalisms [58–62]. In this instance, conduction is achieved through the hopping between localized "Stark states," and the Stark representation (hopping regime) provides an advantageous way to characterize it [63–65].

An interesting example is the variably spaced semiconductor superlattice (VSSL), which is designed to obtain a resonant tunneling between adjacent quantum-well states at a specific value of the growth-direction applied electric field. Such a heterostructure was originally proposed by Summers and Brennan [66] to provide high-energy injection of electrons into a bulk semiconductor substrate at the operating bias and was thoroughly investigated from both theoretical [67–70] and experimental [71–73] points of view. Of special interest is the application of resonant heterostructures in photo-voltaic power generation. In that respect, Barnham and Duggan [74] proposed the use of a VSSL within a *p-i-n* structure to minimize the electron–hole recombination. More recently, Courel et al. [75,76] theoretically investigated the conversion efficiency of *p-i-n* $GaAs/Ga_{1x}In_xN_yAs_{1y}$ solar cells based on resonant tunneling and an improvement of 4% was achieved with respect to standard MQW solar cells. Similar studies in p-i-n $GaAs/Ga_{1x}In_xN_yAs_{1y}$ MQW solar cells were theoretically [77, 78] and experimentally [79] performed. The combined effect of applied electric and magnetic fields has proven to be a powerful tool in the study of several phenomena related to vertical transport in semiconductor heterostructures. Investigations of the influence of in-plane magnetic fields on the negative differential conductivity of double-barrier resonant heterostructures [80] and miniband transport in semiconductor superlattices under magnetic fields [81,82] were carried out at the end of the past century. Nowadays similar studies are of great importance in fields such as laser physics [83,84] and spintronics [85], to mention some few examples. In most of these studies of magnetic-field effects on the electrical and optical properties of resonant heterostructures, one observed a dramatic change in the photoluminescence (PL) spectrum as a function of the applied magnetic field. Similar results for the intraband absorption coefficient of VSSLs under crossed electric and magnetic fields were recently reported [86].

The interband optical absorption spectra of a $GaAs/Ga_{1-x}Al_xAs$ variably spaced semiconductor super-lattice under crossed in-plane magnetic and growth-direction applied electric fields are theoretically investigated. The electronic structure, transition strengths and interband absorption coefficients are analyzed within the weak and strong magnetic-field regimes. A dramatic quenching of the absorption coefficient is observed, in the weak magnetic-field regime, as the applied electric field is increased, in good agreement with previous experimental measurements performed in a similar system under growth-direction applied electric fields. A decrease of the resonant tunneling in the superlattice is also theoretically obtained in the strong magnetic-field regime. Moreover, in this case, we found an interband absorption coefficient weakly dependent on the applied electric field. Present theoretical results suggest that an in-plane magnetic field may be used to tune the optical properties of variably spaced semi-conductor superlattices, with possible future applications in solar cells and magneto-optical devices [87].

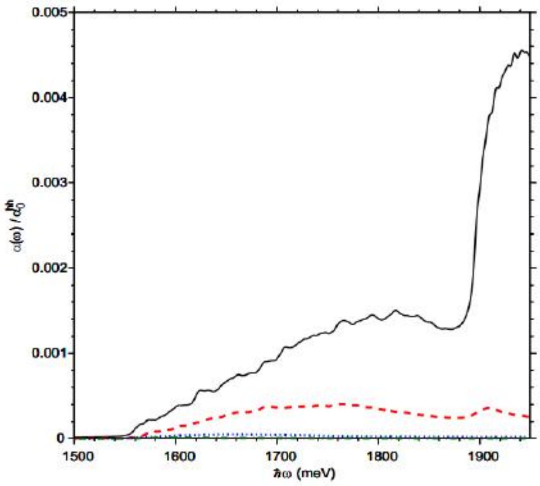

Fig.7. Total interband absorption coefficient as a functions of the photon energy $\hbar\omega$ in the GaAs–$Ga_{0.7}Al_{0.3}As$ variably spaced semiconductor superlattice for B=3 T and different values of the applied electric field. Solid, dashed, dotted, and dot-dashed lines correspond to applied electric fields F=0, F=10 kV/cm, F=20 kV/cm, and F=25 kV/cm, respectively [87].

A theory that considers the electron-phonon interaction in semiconductor superlattices under a magnetic field is proposed for the absorption of light by free charge carriers. The oscillations of the coefficient of light absorption by free charge carriers in the case of scattering by polar phonons were determined, depending on the resonance condition, the frequency of the incident light, and the magnetic field induction, within the framework of the second-order excitation theory, when the magnetic field is directed perpendicular to the surface of the semiconductor superlattice [88].

Density functional theory simulations were utilized to investigate the impact of stacking periodicity on the electrical and optical properties of the GaAs/AlAs superlattice [89]. The observation of the magnetic quantum ratchet effect in graphene with a lateral dual-grating top gate superlattice was reported [90]. In [91], the electrical and optical characteristics of the $InAs/InAs_{0.625}Sb_{0.375}$ superlattices were computed using the relativistic density functional theory paradigm.

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