ENERGY SPECTRUM OF ELECTRONS CONFINED TO A PARABOLIC QUANTUM WELL WITH CONICAL DISCLINATION AND RASHBA SPIN-ORBIT INTERACTION

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The present work investigates the effect of topological defects in quantum dots on the effective g*- factor. Using the energy spectrum, we derived the exact analytic expression for the effective g*-factor and studied the variation of the effective g*-factor with the disclination parameter. It was shown that, in the case of n=0,l=-1, as the magnetic field increases, the effective g*-factor changes sign and increases, approaching the value n=0,l=0. In the case of n=0,l=1, it decreases and approaches n=0,l=0. In the case of n=0,l=0, the effective g factor remains constant as the magnetic field changes.

Keywords: effective g*-factor, quantum dot,disclination.
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1. INTRODUCTION

In recent years there has been increased interest, both theoretically and experimentally in quantum dot geometry in the presence of a magnetic field. Since nanostructures have been applied to electronic devices, interest in them has increased. The reason for this is that in quantum dots there are observed interference effects under the influence of electromagnetic potentials, known as Aharonov-Bohm [1] and Aharonov-Casher [2] effects which have no analog in classical physics.

The paper [3] studied the changes introduced by the linear magnetic flux on the energy spectrum of a free particle confined to move between two cylindrical concentrically shells in a space with a linear defect, that is the combination of a disclination and a dislocation which is called dispiration.

Linear and nonlinear optical properties of a GaAs quantum dot (QD) confined by a parabolic plus inverse square potential with a disclination were theoretically studied [4] under the influence of a magnetic field and the Aharonov-Bohm (AB) flux field.

In [5], the influence of topology in quantum dynamics in two-dimensional quantum dots in a conic surface. The authors analyzed the quantum dynamics of particles in this dot when submitted to an external magnetic field and Aharonov-Bohm flux in the dot center. The paper's author studied [6] a 2D mesoscopic dot with an anisotropic effective mass considering surface quantum confinement effects. Consider that the dot is defined on the surface of a cone, which can be controlled topologically and mapped to the 2D dot in flat space. Afterward demonstrate through numerical analysis that the electronic properties, the magnetization, and the persistent current undergo significant changes due to quantum confinement and non-isotropic mass. In the work [7] authors report the equivalence of the geometries of a cylindrical shell with screw dislocation and another without defect but with a larger radius.

Topological defects are the defects in the system that cannot be removed by smooth continuous deformation [8]. Topological defects can be the source of changes in a material's electrical, optical, or magnetic properties. In [9] they have shown that screw dislocations make the host semiconducting nanocrystals essentially chiral and optically active.

Katanaev and Volovich’s [10] approach which translates the theory of defects in solids into the three-dimensional language. Wedge dislocations are rare in Nature because they require a large amount of medium to be added or removed, which results in a large expenditure of energy. From the qualitative standpoint, creating a wedge dislocation is equivalent to introducing a conical singularity[11,12].

In the paper [13], the authors examined the effect of introducing a conical disclination on the thermal and optical properties of a two-dimensional GaAs quantum dot in the presence of a uniform and constant magnetic field. In particular, using the model consists of a single-electron subject to a confining Gaussian potential with a spin-orbit interaction in the Rashba approach.

The work [14] obtained the modifications to the traditional Landau-Fock-Darwin spectrum in the presence of conical disclination. The effect of the conical kink on the degeneracy structure of the energy levels was investigated.

The interaction of electron states with the lattice potential in nanocrystals leads to the renormalization of the g-factor [15,16].

The nonsimply connected topology of the quantum dots has been attracting for a long time the careful attention of physicists, chemists, and mathematicians.

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In this paper, we consider semiconductor two dimensional quantum dot with a topological defect given by a conical disclinations and Rashba spin-orbit interactions and investigate the effect of disclination on the effective g^*-factor of electrons. We use the Volterra design [10] to model the ring wedge dislocation defect of radius R and the remote wedge dislocation. The procedure to create a wedge dislocation is obtained by either removing (positive-curvature wedge dislocation) or inserting (negative-curvature wedge dislocation) an angle $2\pi|\alpha - 1|$ such that the total angle around the z-axis is $2\pi\alpha$ instead of 2π.

In this paper, we consider the influence of the Rashba spin-orbit interaction and wedge disclination in the effective g*-factor in InSb type quantum ring.

2. ENERGY SPECTRUM FOR A DISCLINATED QUANTUM RING WITH RASHBA SPIN-ORBIT INTERACTION

We consider the quantum ring with a wedge dislocation in an external magnetic field. Conical disclination modifies the metric of a ring from its otherwise Euclidean
form as given below:

$$ds^2 = d\rho^2 + \rho^2 d\theta^2 \tag{1}$$

We consider one wedge dislocation in semiconductor quantum dot. For this, we perform one more coordinate transformation [6]:

$$\rho = \alpha r, \phi = \frac{\theta}{\alpha} \tag{2}$$

The metric (1) has the form

$$ds^2 = \alpha^{-2} dr^2 + \rho^2 d\phi^2 \tag{3}$$

This is more frequently used form of the metric for a conical singularity [12].

Where describes a conical surface for $\rho \geq 0$ and $0 \leq \theta < 2\pi$, describes a conical surface. For $0 < \alpha < 1$ (deficit angle), the metric (1) describes an actual cone. The total Hamiltonian of the system is given by:

$$H = \frac{1}{2m}\left(\vec{P} + e\vec{A}\right)^2 + V_c\left(\alpha, r\right) + H_{RSOI} + H_Z \tag{4}$$

where m is the effective mass of the electrons, and $\vec{A}$ is the vector potential.

The parabolic potential for the quantum dot as

$$V_c(\alpha, \mathrm{r}) = \frac{m\omega^2}{2\alpha^2} r^2 \tag{5}$$

For a uniform magnetic field parallel to the z-axis, the vector potentials in cylindrical coordinates have the components $A_r = 0, A_\theta = \dfrac{Hr}{2\alpha^2}, A_z = 0$, The Rashba spin-orbital term has the form as [15]

$$H_{RSOI} = \gamma\sigma_z \alpha \frac{dV_c}{dr}\left(-i\frac{1}{r}\frac{\partial}{\partial\theta} + \frac{eHr}{2\hbar\alpha^2}\right) \tag{6}$$

Where σ_z is the Pauli matrix and γ is the Rashba spin-orbit coupling parameter

The Hamiltonian in cylindrical coordinates is given by

$$\begin{aligned} H = &-\frac{\hbar^2}{2m} I\left[\alpha^2 \frac{\partial}{r\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{r^2\partial\theta^2}\right] - i\frac{I}{2}\frac{\hbar\omega_c}{\alpha^2}\frac{\partial}{\partial\theta} + \frac{I}{2}m\omega^2\alpha^{-2}r^2 + \frac{I}{8}m\omega_c^2\alpha^{-4}r^2 \\ &-\gamma\sigma_z \frac{m\omega^2 r}{\alpha}\frac{i}{r}\frac{\partial}{\partial\theta} + \gamma\sigma_z\frac{m\omega^2 r^2}{\alpha^3}\frac{\omega_c}{2\hbar} + \frac{\sigma_z}{4} g\hbar\omega_c \frac{m}{m_0} \end{aligned} \tag{7}$$

where $\omega_c = \dfrac{eB}{m}$ is the electronic cyclotron frequency and m_0 is the free-electron mass, I is a 2x2 unit matrix.

The wave function of an electron has the form [12,15]:

$$\Psi\left(r, \theta\right) = e^{il\theta} R_{n,l,\sigma}(\mathrm{r}) \tag{8}$$

For the radial function, we find the following equation:

$$\begin{aligned} &I\left[\frac{\partial}{r\partial r}\left(r\frac{\partial}{\partial r}\right) - \frac{l^2}{\alpha^2 r^2}\right] R_{n,l,\sigma}(\mathrm{r}) - \frac{1}{\hbar^2\alpha^4}\frac{I\Omega^2}{4} m^2 r^2 R_{n,l,\sigma}(\mathrm{r}) + \\ &\frac{2m}{\hbar^2\alpha^2} I\left(E - \frac{1}{2} l \frac{\hbar\omega_c}{\alpha^2} - \gamma\sigma_z \frac{m\omega^2}{\alpha} l - \frac{\sigma_z}{4} g\hbar\omega_c \frac{m}{m_0}\right) R_{n,l,\sigma}(\mathrm{r}) = 0 \end{aligned} \tag{9}$$

where $\Omega = \sqrt{4\omega^2 + 4\gamma\sigma_z \frac{m}{\alpha}\frac{\omega_c}{\hbar} + \omega_c^2\alpha^{-2}}$

The eigenvalue

$$E_{nl} = \left(2n+1+\frac{|l|}{\alpha}\right)\hbar\Omega + \frac{1}{2}\frac{\hbar\omega_c l}{\alpha^2} + \frac{\sigma}{2} g^* \mu_B H + \frac{\gamma m\omega^2 l}{\alpha}\sigma \tag{10}$$

This formula differs from the formula in work number [11]: (15b) by the denominator α in the last term. The reason for this difference is that the vector potential term in the Rashba term is taken as different from the potential term $\frac{1}{2m}\left(\vec{P}+e\vec{A}\right)^2$ in work [12]. In both terms the same vector potential must be used. The radial wave function is given by

$$R_{n,l,\sigma}(\mathrm{r}) = Ce^{-\frac{1}{2\alpha^2}\frac{r^2 m\Omega}{\hbar}}\left(\frac{r^2 m\Omega}{\alpha^2\hbar}\right)^{\frac{|l|}{2\alpha^2}} L_n^{\frac{|l|}{\alpha}}\left(\frac{r^2 m\Omega}{\hbar}\right) \tag{11}$$

The effective g^*-factor can be determined from the Zeeman splitting of subbands:

$$g^* = \frac{E_\uparrow - E_\downarrow}{\mu_B H} \tag{12}$$

Here $E_\uparrow$ and $E_\downarrow$ are the electron energy for spin +z and −z directions, respectively.

Let us investigate the effect of the Rashba spin-orbit interaction on the spin splitting of electrons in quantum dots with wedge disclinations. Figure 1 shows the variation of the effective g-factor of electrons in InSb-type quantum dots in the presence of Rashba spin-orbit interaction and wedge dislocations depending on the orbital quantum number l at n=0. As seen from Figure 1 The effective g^*-factor of electrons in InSb-type quantum dots in the presence of Rashba spin-orbit interaction and wedge dislocations as functions of orbital quantum number l is of the step type. We

Introduce a parameter $y = \frac{\omega_c}{\omega}$ to quantify the magnetic field strength, and $r = \sqrt{\frac{\hbar}{m\omega}}$ =148 $\mathring{A}$. We use the following material constants: $g_0 = 3.2$ for InSb quantum dot $\gamma = 500\,\mathring{A}^2$ [17].

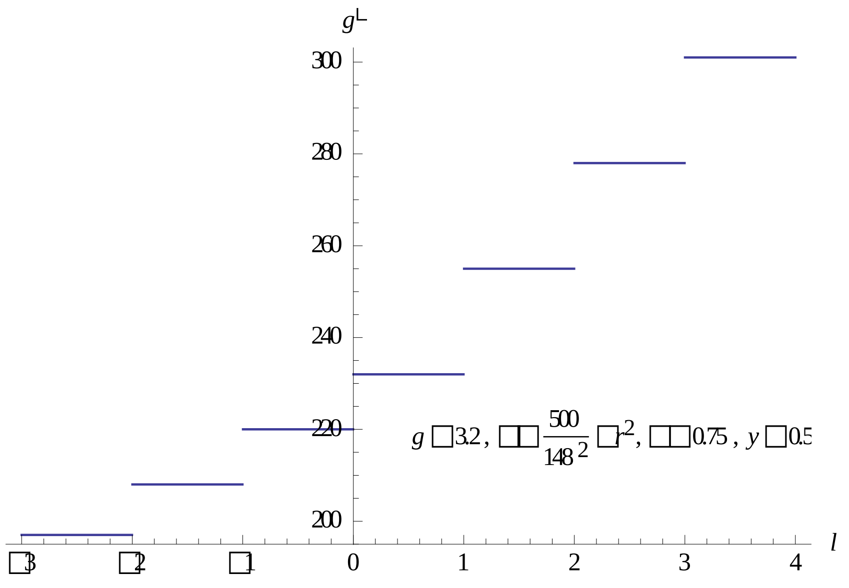

Fig. 1. The variation of the effective g^*-factor of electrons in InSb-type quantum dots in the presence of Rashba spin-orbit interaction and wedge dislocations depending on the orbital quantum number l at n=0.

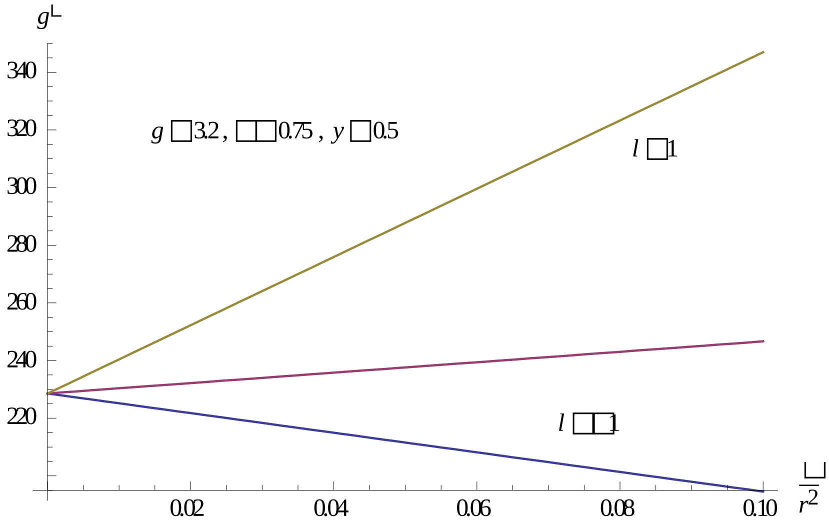

Fig.2. The dependence of the effective g^*-factor of the electrons in InSb type quantum dots on the dimensionless Rashba parameter at n=0,l=-1,0,1 for y=0.5.

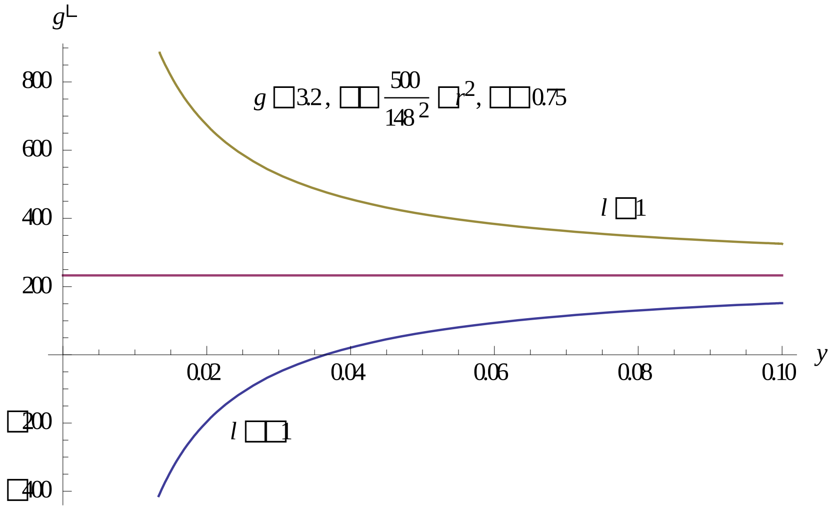

Fig.3. The dependence of the effective g^*-factor of the electrons in InSb type quantum dots on the magnetic field parameter y at n=0,l=-1,0,1 for α=0.75.

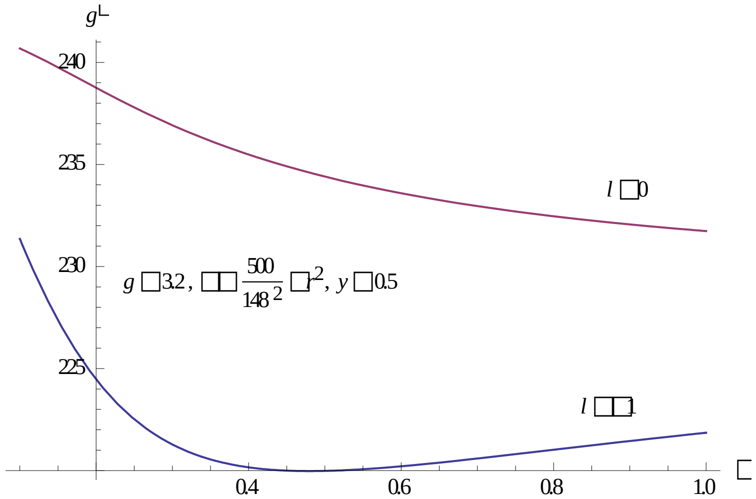

Fig.4. The dependence of the effective g^*-factor of the electrons in InSb type quantum dots on the disclination parameter α at n=0,l=-1,0 for y=0.5.

The height of the step is not one but 11.89 for negative quantum number l, and for positive l 22.89, unlike the case where the magnetic field and Rashba splitting are zero. Fig.2 shows the dependence of the effective g-factor of the electrons in InSb-type quantum dots on the dimensionless Rashba parameter at n=0,l=-1,0,1 for y=0.5.

As can be seen from Figure 2, for quantum numbers l = 0,1 as the Rashba parameter increases, the effective g factor increases, and for l = -1 it decreases. In Fig.2 we plot the dependence of the effective g^*-factor of the electrons in InSb-type quantum dots on the magnetic field parameter y= at n=0,l=-1,0,1 for α=0.75.

As can be seen from the figure, in the case of n=0,l=-1, as the magnetic field increases, the effective g-factor changes sign and increases, approaching the value n=0,l=0. In the case of n=0,l=1 it decreases and approaches n=0,l=0. In the case of n=0,l=0, the effective g factor remains constant as the magnetic field changes. In Fig.4 we plot the dependence of the effective g^*-factor of the electrons in InSb-type quantum dots on the disclination parameter α at n=0,l=-1,0 for y=0.5.

As can be seen from the figure, at n=0,l=-1 the effective g^*- factor decreases as the disclination parameter increases, it reaches the minimum value and then increases. At n=0,l=1 the effective g factor decreases.

3. CONCLUSIONS

We present a theoretical study of the effect of Rashba spin-orbit interaction and conical disclination on the effective g-factor of a type quantum dot. The dependence of the effective g^*-factor on the parameters of the quantum dot and the applied external magnetic field is studied. It has been shown that, at the value of quantum number n=0,l=-1 the effective g factor decreases as the disclination parameter increases, it reaches the minimum value and then increases. At n=0,l=1 the effective g^*-factor decreases.

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