THE MAGNETIC SUSCEPTIBILITY OF ELECTRONS IN QUANTUM RINGS WITH RASHBA SPIN-ORBIT INTERACTION IN A WEAK MAGNETIC FIELD

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We investigated the magnetic properties of electrons in quantum rings with Rashba spin-orbit interaction and Zeeman splitting in a weak magnetic field. It was shown that the magnetic susceptibility is a periodic function of the AB flux and changes from a negative value to a positive value at the Rashba spin-orbit coupling parameter ξ = 0.02.But the value ξ=2 varies from positive to negative with changes in the AB flux and amplitude oscillations increase.

Keywords: semiconductor ring, partition function, Rashba spin-orbit interaction.
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1. INTRODUCTION

The rapidly developing field of spintronic is a field in which new electronic devices are created using the spin degrees of freedom of electrons. Spin-orbit interaction (SOI) in semiconductors is a central mechanism that determines the fundamental physics in low-dimensional systems and is largely responsible for the possibility of semiconductor structures as potential quantum devices. Rashba spin-orbit interaction (RSOI) is the one typical spin-orbit interaction that one encounters in a conventional semiconductor. The Rashba spin-orbit interaction in semiconductors is caused by an electric field resulting from a structural inversion asymmetry. In recent years [1, 2], the development of the field of spintronics in semiconductors has triggered the investigation of two-dimensional (2D) electron systems with spin-orbit interaction.

The emergence of quantum effects- known as the Aharonov-Bohm (AB) [3] and Aharonov-Casher (AC) [4] effects in quantum rings, which are unique in classical physics, has attracted experimental and theoretical interest in quantum rings. The Aharonov-Bohm (AB) effect occurs in an experiment with an electron beam embracing an infinitely thin solenoid that bears the magnetic flux Φ. The scattering cross-section of the electrons periodically depends on the magnetic flux Φ [5]. In 1984, Aharonov and Casher (AC) discovered the dual of the AB effect: a neutral particle with a magnetic moment encircling a charged line accumulates the AC phase. Aharonov and Casher's effect is due to the interaction between the particle's magnetic moment μ and an electric field E.

The influence of Rashba spin-orbit coupling on zero conductance resonances appearing in one-dimensional conducting rings asymmetrically coupled to two leads was investigated in [6]. The transmission amplitude through an asymmetric Aharonov–Bohm ring was derived analytically [7], in the presence of the Rashba spin-orbit interaction.

The authors [8] derived an exact expression for the zero-temperature conductance of a one-dimensional ring connected to two leads in the presence of spin-orbital interaction. The authors of the paper [9] investigated the Aharonov-Casher effect in a mesoscopic ring in the presence of a cylindrically symmetric electric field.

In [10] was analyzed the zero-conductance resonances appearing in an AB and AC ring as a signature of interfering resonant states of the loop system under the influence of a magnetic flux and a Rashba electric field in the presence of a tunable tunnel barrier. In the paper [11] investigated the ballistic transport through a 1D ring, symmetrically connected to two leads, in the presence of a magnetic field and of Rashba and Dresselhaus spin-orbital terms.

In the paper [12] considered the lattice of quantum rings and investigated the magnetic properties using the 2D rotator model. In this paper, we consider the influence of Rashba spin-orbit interaction on the magnetization, of a quantum ring in a vertical magnetic field. For this purpose, we obtain the spectrum and the eigenstates of electrons, then we calculate the partition function of the system.

2. THEORY

In this paper, we investigate the influence of the Rashba spin-orbit interaction on the magnetic moment of electrons in a quantum ring in a weak magnetic field. We consider a one-dimensional quantum ring of radius R in the presence of electric and magnetic field. The electric field originates from an asymmetric confinement along the z-direction and is perpendicular to the plane of the ring; the magnetic field is in the radial direction. The Hamiltonian of 2D electrons in an external magnetic field and Rashba SOI have

$$H = \frac{1}{2m}\left(\vec{P} - e\vec{A}\right)^2 + \frac{\alpha_R}{\hbar}\left[\vec{\sigma}\times\left(\vec{P} - e\vec{A}\right)\right]_z \quad (1)$$

In this article, we used a system of units in which the speed of light is c = 1. For a uniform magnetic field parallel to the z-axis, the vector potentials in cylindrical coordinates have the components

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$A_\phi = \frac{H\rho}{2}, A_\rho = 0, A_z = 0$ and the Hamiltonian for the one-dimensional quantum ring as [7, 8]:

$$H = \hbar\Omega\left(-i\frac{\partial}{\partial\varphi} + \frac{\omega_{so}}{2\Omega}\sigma_r - \frac{\Phi}{\Phi_0}\right)^2 \tag{2}$$

Where $\Omega = \frac{\hbar}{2ma^2}, \omega_{so} = \frac{\alpha}{\hbar a}$

$\sigma_r = \sigma_x \cos\varphi + \sigma_y \sin\varphi, \Phi = \pi R^2 H$ is the magnetic flux through a quantum ring, and $\Phi_0 = \frac{2\pi\hbar}{e}$ is the magnetic flux.

The eigenvalue problem was solved in [9] and the energy eigenvalues are:

$$E_{n\sigma} = \varepsilon_0\left(n - \frac{\Phi_{AB}}{2\pi} - \frac{\Phi_{AC}^\sigma}{2\pi}\right)^2 \tag{3}$$

where $\Phi_{AC}^\sigma = -\pi\left(1 + (-1)^\sigma\sqrt{1+\xi^2}\right)$ is the so-called Aharonov-Casher phase, $n = 0, \pm 1, \pm 2, \ldots$ is the azimuthal quantum number. $\varepsilon_0 = \frac{\hbar^2}{2mR^2} = \hbar\Omega, \Phi_{AB} = 2\pi\frac{\Phi}{\Phi_0} = 2\pi\,\mathrm{f}, \xi = \frac{2mR\alpha}{\hbar^2}$

$\sigma = \pm 1$ is the spin quantum number.

When the Zeeman term is present, the interaction between the electron spin and a relatively *weak* magnetic field B can be treated by perturbation theory. The Zeeman term

$$H_{Zp} = \sigma_z g\hbar\Omega\Phi_{ab}\frac{m}{m_0} \tag{4}$$

is the perturbation of the Hamiltonian (2),m_0 -is the free electron mass.

In the first-order approximation one neglects the off-diagonal elements; we obtain the energies, including the first-order corrections [10]:

$$E_{n\sigma} = \varepsilon_0\left(n - \frac{\Phi_{AB}}{2\pi} - \frac{\Phi_{AC}^\sigma}{2\pi}\right)^2 - (-1)^\mu\frac{gm}{m_0}\Phi_{AB}\cos\theta \tag{5}$$

Where

$$\theta = 2\arctan\left(\frac{\Omega - \sqrt{\Omega^2 + \omega_{so}^2}}{\omega_{so}}\right) \tag{6}$$

The no interacting quantum ring is assumed to be in equilibrium with a heat reservoir at temperature T. The starting point of the thermodynamic analysis is the evaluation of the partition function for the energy spectra given in Eq. (5):

$$Z = \sum_{n\sigma} e^{-\beta E_{n\sigma}} \tag{7}$$

Where $E_{n\sigma}$ is the energy spectrum of the considered system, $\beta = (k_B T)^{-1}$ and k_B is the Boltzmann constant and T is the thermodynamic equilibrium temperature.

Then, after summation over n the expression in (7) becomes

$$Z = \sqrt{\pi\chi}\sum_\sigma e^{(-1)^\sigma\frac{\Phi_{AB}}{\chi}g\cos\vartheta} v_3\left(-\pi P^\sigma, e^{-\pi^2\chi}\right) \tag{8}$$

where $v_3\left(-\pi P^\sigma, e^{-\pi^2\chi}\right)$ is the Elliptic theta function, $\chi = (\beta\varepsilon_0)^{-1}$ and $P^\sigma = \frac{\Phi_{AB}}{2\pi} + \frac{\Phi_{AC}^\sigma}{2\pi}$. The free energy of a nondegenerate system can be obtained using the partition function as

$$F = -k_B T\ln Z = -\chi\varepsilon_0\ln\left(\sqrt{\pi\chi}\sum_\sigma e^{(-1)^\sigma\frac{\Phi_{AB}}{\chi}g\cos\vartheta} v_3\left(-\pi P^\sigma, e^{-\pi^2\chi}\right)\right) \tag{9}$$

The magnetization of the electron gas can be written as:

$$M = -\frac{\partial F(\chi, f)}{\partial H} \tag{10}$$

Let us define the magnetic susceptibility

$$q = \frac{\partial M}{\partial H} \tag{11}$$

For our calculation, we consider the parameter corresponding to InSb materials: m = 0.014m0, where m0 is the free electron mass, g = 3.2,α = 500A^2, taken from the literature [13].

Fig.1 and Fig.2 show the variation of magnetic susceptibility (per unit of $\left(\frac{2\varepsilon_0}{\mu_B^2}\right)$)) on the AB flux at fixed temperature χ=0.05, for the Rashba parameters ξ=0.02, ξ=1. We see that the magnetic susceptibility is a periodic function of the AB flux and changes from a negative value to a positive value at the Rashba spin-orbit coupling parameter ξ = 0.02. When the Rashba parameter ξ = 1, the magnetic susceptibility changes from a positive value to a negative value.

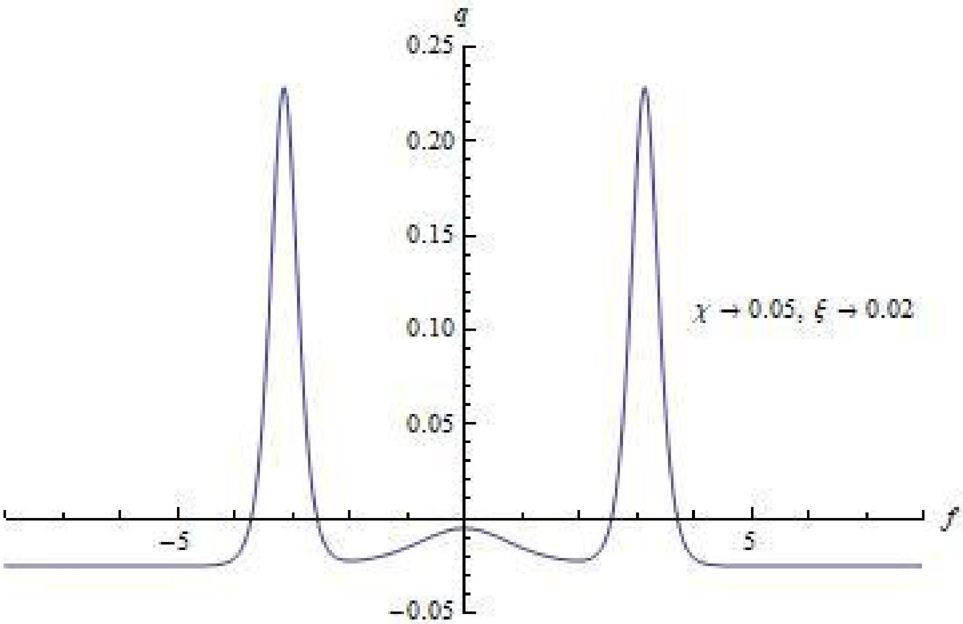

Fig. 1. The variation a magnetic susceptibility (per unit of $\left(\frac{2\varepsilon_0}{\mu_B^2}\right)$)) on the AB flux at fixed temperature χ=0.05, for the Rashba parameters ξ=0.02.

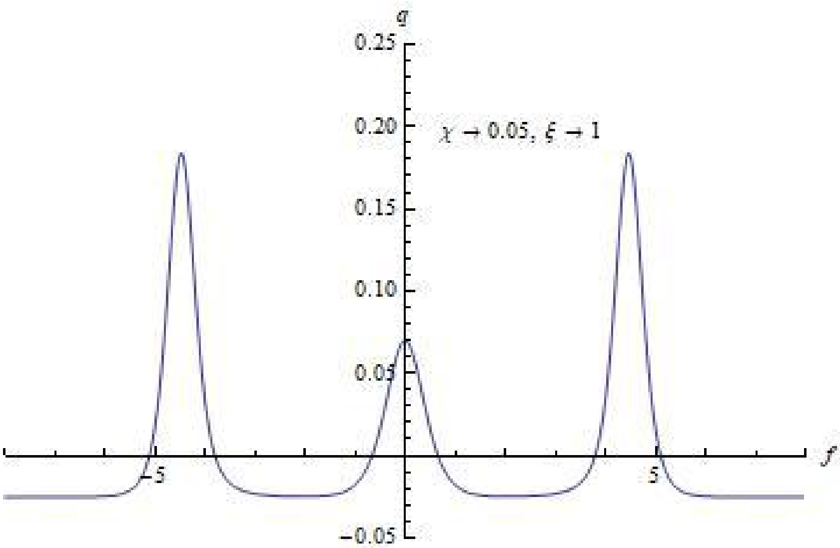

Fig. 2. The variation of magnetic susceptibility (per unit of $\left(\frac{2\varepsilon_0}{\mu_B^2}\right)$)) on the AB flux at fixed temperature χ=0.05, for the Rashba parameters ξ=1.

3. CONCLUSIONS

We have studied the magnetic properties of electron gas on quantum rings in the presence of the externally applied static magnetic field in the presence of Rashba spin-orbital interaction terms by using the canonical ensemble approach. It was shown that in the presence of Rashba spin-orbital interaction the magnetic susceptibility varies from positive to negative values with the change of the magnetic flux at a fixed temperature.

[1] *R. Winkler*. Spin-Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems (Berlin: Springer) 2003.
[2] *D. Bercioux and P. Lucignano.* Quantum transport in Rashba spin-orbit materials: a review Report on Progress in Physics, 78, 2015, 106001.
[3] *Y. Aharonov and D. Bohm.* Phys. Rev. 115, 485, 1959.
[4] *Y. Aharonov and A. Casher.* Phys. Rev. Lett. 53, 319, 1984.
[5] *A.V. Chaplik, Vadim Kovalev*. New Versions of the Aharonov-Bohm Effect in Quantum Rings, Article in NanoScience and Technology · 2014 DOI: 10.1007/978-3-642-39197-2_9
[6] *Urs Aeberhard, Katsunori Wakabayashi and Manfred Sigrist.* Phys.Rev B 72, 075328, 2005.
[7] *D.M. Baltateanu.* 120 Acta Physica Polonica A № 6, 1047, 2011.
[8] *B. Molnar and F. M. Peeters, P. Vasilopoulos.* Phys.Rev B 69,155335, 2004.
[9] *Sangchul Oh and Chang-Mo Ryu*. Phys. Rev. B 51, 13441, 1995.
[10] R. Citro, F. Romeo, and M. Marinaro. Phys. Rev B 74, 115329, 2006.
[11] *P. Vasilopoulos, X.F. Wang.* Physica E 34 (2006) 359–362.
[12] *Peter A. Meleshenko, Alexander F. Klinskikh.* Journal of Magnetism and Magnetic Materials323 (2011), 2663–2668.
[13] *O. Voskoboynikov, O. Bauga, C.P. Lee and O. Tretyak.* (2003) Magnetic properties of parabolic quantum dots in the presence of the spin-orbit interaction, Journal of Applied Physics, 94 (9), 5891.

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