DEPENDENCE OF SIGNAL PULSE ENERGY DENSITY ON THE CHARACTERISTIC LENGTHS IN METAMATERIAL

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Set of nonstationary truncated equations is solved to calculate spectral density and hence energy density of signal pulse. The pumping pulse is considered to be Gaussian. Dependences of energy density on the characteristic lengths were depicted. It is shownthat the energy density has clearly maxima versus characteristic lengths. In addition, it was observed that the energy density decreases with increase in phase modsulation parameter of idler pulse. In contrast to ordinary nonlinear medium larger values of energy are obtained at the input, but not at the output of metamaterial.

Keywords: nonstationary frequency conversion, group velocity difference, group velocity dispersion, Gaussian pulse, metamaterial, dispersion propagation length, dispersion propaation time.
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1. INTRODUCTION

The interest in non-stationary optical processes has increased dramatically since the development of lasers that generate ultrashort pulses and have synchronized longitudinal modes. Generation of second harmonics of frequencies of ultrashort pulses emitted by ruby and neodymium lasers is described by the first approximation of dispersion theory [1]. In order to convert the frequencies of coherent optical radiation, along with the acquisition of harmonics, frequency mixing processes are also used. A system of non-stationary truncated equations is applied for the analysis of the mixing of ultrashort pulses with a strong pumping wave. The interest in nonlinear frequency mixing is related to the practical realization of frequency-tunable UV radiation, which is achieved by changing the synchronism angle in a nonlinear crystal. Frequencies that can be synchronously mixed are selected from a wide spectrum of radiation. For the broadband spectrum, the dispersion of the nonlinear medium is taken into account in the second order dispersion theory. In the first approximation of the dispersion theory, only the effects related to the difference in group velocities are taken into account. Taking into account the difference in group velocities with the application of constant intensity approximation (CIA), the second harmonic generation (SHG) process has been studied in detail for the case of ordinary nonlinear media [2].

Amplification of weak signals and expansion of the radiation frequency range of lasers require the creation of optical parametric amplifiers. The interest in creating of powerful sources of light pulses with a duration measured in femtoseconds is due to the non-stationary interaction of ultrashort pulses.

The first theoretical studies of such parametric amplifiers was conducted by the authors [3]. The character of the interaction of modulated ultrashort pulses depends on the dispersion properties of the medium [4]. Metamaterials with a negative refractive index at certain frequencies attract attention with their unique characteristics of interaction with electromagnetic waves. The dynamics of the generation of the second harmonic during the three-wave interaction in metamaterials was considered [5]. Metal-based negative refractive index metamaterials have been extensively studied in the microwave region of the spectrum. However, these materials have not been applied in the near-infrared and visible regions of the spectrum due to the difficulty of manufacturing technologies and the poor optical properties of metals at these wavelengths.

The fact that the metal-dielectric-metal triple-layer structure has a negative refractive index around the 2 micrometer wavelength was confirmed for the first time and the results can be used for the application of this metamaterial composite in the infrared and visible regions received by the authors [6, 7].

The dye-based lasers with a wide frequency range are used to receive UV radiation whose frequency can be adjusted [8]. Using such lasers as the main radiation source changes the frequency conversion conditions, i.e., the converted UV radiation falls into the strong dispersion region of the KDP crystal, which is more suitable for those experiments. Since the dispersion length related to the propagation of pulses due to dispersion is smaller than the length of the applied nonlinear crystals, the necessity of using the second approximation of the dispersion theory emerges. Generation of second harmonic, generation of sum frequency, etc. the non-stationarity of the processes is caused by the group delay and dispersion effects of radiation pulses.

These effects are observed for pulses whose duration is smaller than picoseconds. That is, the

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duration of the pulse should be hundreds of times less than the time it passes through its medium (for instant, nonlinear crystal, metamaterial, etc. However, in the process of parametric generation of light, the duration of the pulse generation, that is, the period of generation of parametric oscillations, can be the same as the duration of the pumping pulse [9]. If the duration of the pumping pulse $\tau_{duration}$ is shorter than the new pulse generation time $\tau_{creation}$ then the parametric generation of light cannot occur. Parametric oscillations occur when $\tau_{duration} \succ \tau_{creation}$ and parametric generation occurs in the quasi-stationary regime.

2. DISCUSSIONS AND RESULTS

Assume, that mutual interaction occurs when the pump and idler waves propagate in the positive direction of the choozen z-axis $(z = 0)$, while the signal wave in the opposite direction $(z = 0)$. During such a parametrical interaction the energy exchange occurs between the wave packets of the pump $A_2(t,z)$ idler $A_3(t,z)$ and signal $A_1(t,z)$ waves which results in parametrical amplification of the signal wave. Calculations take into account, that dielectric permittivity and magnetic permeability of metamaterial simultaneously receive negative values at signal frequency ω_1, and positive values at frequencies ω_2 and ω_3 of the pump and idler waves respectively. Three frequency interaction in the second order dispersion theory is analized by the following set of reduced equations [9]:

$$\left(\frac{\partial}{\partial z} - \frac{1}{u_1}\frac{\partial}{\partial t} - i\frac{g_1}{2}\frac{\partial^2}{\partial t^2} - \delta_1\right)A_1 = i\gamma_1 A_3 A_2^* e^{i\Delta z}$$

$$\left(\frac{\partial}{\partial z} - \frac{1}{u_2}\frac{\partial}{\partial t} - i\frac{g_2}{2}\frac{\partial^2}{\partial t^2} - \delta_2\right)A_2 = i\gamma_2 A_3 A_1^* e^{i\Delta z} \qquad (1)$$

$$\left(\frac{\partial}{\partial z} - \frac{1}{u_3}\frac{\partial}{\partial t} - i\frac{g_3}{2}\frac{\partial^2}{\partial t^2} - \delta_3\right)A_3 = i\gamma_3 A_1 A_2 e^{-i\Delta z}$$

where A_j $(j = 1-3)$ – are the complex amplitudes of the signal, pump and idler waves respectively, δ_j – - are the absorption coefficients of those waves, u_j – refer to the group velocities, $\Delta = k_1 - k_2 - k_3$ – is the difference in wave numbers (the phase mismatch parameter), $g_j = \partial^2 k_j / \partial\omega_j^2$ – are the group velocity dispersion coefficients, γ_1, γ_2, γ_3 – are the coefficients of nonlinear coupling.

If assume, that amplitude of the pumping pulse remains unchangable $(A_2 = A_{20} = const.)$ then from equation (1) we get:

$$\left(\frac{\partial}{\partial z} - \frac{1}{u_1}\frac{\partial}{\partial t} - i\frac{g_1}{2}\frac{\partial^2}{\partial t^2} - \delta_1\right)A_1 = i\gamma_1 A_3 A_2^* e^{i\Delta z}$$

$$\left(\frac{\partial}{\partial z} - \frac{1}{u_3}\frac{\partial}{\partial t} - i\frac{g_3}{2}\frac{\partial^2}{\partial t^2} - \delta_3\right)A_3 = -i\gamma_3 A_1 A_2 e^{-i\Delta z} \qquad (2)$$

The set above in local time variable $\eta = t - z/u_1$ is presented by:

$$\left(\frac{\partial}{\partial z} - i\frac{g_1}{2}\frac{\partial^2}{\partial \eta^2} - \delta_1\right)A_1(z,\eta) = i\gamma_1 A_3 A_2^*(z,\eta) e^{i\Delta z}$$

$$\left(\frac{\partial}{\partial z} - \nu\frac{\partial}{\partial t} - i\frac{g_3}{2}\frac{\partial^2}{\partial \eta^2} - \delta_3\right)A_3 = -i\gamma_3 A_1 A_{20}(z,\eta) e^{-i\Delta z} \qquad (3)$$

where $\nu = 1/u_2 + 1/|u_1|$. Applying Fourye transformations yields:

$$\left(\frac{\partial}{\partial z} + i\frac{g_1}{2}\omega^2 - \delta_1\right)A_1(z,\omega) = i\gamma_1 A_3 A_{20}^*(z,\omega) e^{i\Delta z}$$

$$\left(\frac{\partial}{\partial z}+i\frac{g_3}{2}\omega^2+i\nu\omega+\delta_3\right)A_3(z,\omega)=-i\gamma_3 A_1 A_{20}(z,\omega)e^{-i\Delta z} \tag{4}$$

According to the geometry of the task we put boundary conditions

$$A_1(z=l)=0,\ \mathrm{A}_{2,3}(z=0)=A_{20,30} \tag{5}$$

which yields for the complex amplitude of signal pulse:

$$A_1(z,\omega)=\frac{i\gamma_1 A_{30}A_{20}^*}{\lambda-ktg\lambda l}(\sin\lambda z-tg\lambda l\cdot\cos\lambda z)\times\exp\left(\frac{\delta_1-\delta_3}{2}z-i\frac{\omega^2 g_3-\omega\nu-\Delta}{2}z\right) \tag{6}$$

Here $\lambda^2=-\Gamma_2^2-(\delta_1+\delta_3+im)^2/4,\ m=(\omega^2 g_3-\omega\nu-\Delta)/2,\ g=g_3-g_1,\ \Gamma_2^2=\gamma_1\gamma_2 I_{20}$

When there are not losses $(\delta_{1,3}=0)$ in medium, the expression above is simplified:

$$A_1(z,\omega)=\frac{i\gamma_1 A_{30}A_{20}^*(\sin\lambda' z-tg\lambda' l\cdot\cos\lambda' z)}{\lambda'-i\dfrac{\omega^2 g/2-\omega\nu-\Delta}{2}tg\lambda' l}\times\exp\left(-i\frac{\omega^2 g_3-\omega\nu-\Delta}{2}z\right) \tag{7}$$

where $\lambda'=\left[(\omega^2 g/2-\omega\nu-\Delta)^2/4-\Gamma_2^2\right]^{1/2},\ \Gamma_2^2=\gamma_1\gamma_3 I_{20}$

Calculations for the spectral density $S_1(z,\omega)=A_1(z,\omega)\cdot A_1^*(z,\omega)$ and hence for the energy yields

$$W_1=\int_{-\infty}^{\infty}S_1(z,\omega)d\omega$$

Assuming the pump pulse to be Gaussian and having put equation (7) for the energy of signal wave gives:

$$W_1=D\int_{-\infty}^{\infty}\frac{(\sin\lambda' z-tg\lambda' l\cdot\cos\lambda' z)^2}{\lambda'^2+\dfrac{k^2}{4}tg^2\lambda' l}\cdot\exp\left(-\frac{\omega^2\tau^2}{1+C^2\tau^4}\right)d\omega \tag{8}$$

here $D=cn_1\gamma_1^2 I_{30}I_{20}\tau^2/16\pi^2$, $\mathrm{k}=\omega^2\mathrm{g}/2-\omega\nu-\Delta$

The delay of two pulses with the difference of the inverse values of the group velocities ν relative to each other at the output of the medium is called the characteristic delay time: $\tau_\nu=\nu l$; The distance by which the two pulses are delayed relative to each other by time τ_1 is the quasi-static interaction length: $l_\nu=\tau_1/\nu$: If $\tau_1\lhd\tau_\nu$ or $l\rhd l_\nu$, then the process becomes nonstationary.

In the quasi-static approximation, the time difference for the two pulses to pass through the nonlinear medium is assumed to be much smaller than the duration of the filler pulse: $l|1/u_1-1/u_2|\le\tau_1$ or $l\le l_d$.

The last relation shows that the time of passing through the medium is less than the time required for the pulse to be deformed. In other words, the pulse cannot change its form. In a dispersive medium, pulses of different frequencies or Fourier terms, in other words, monochromatic pulses propagate with different phase velocities $c/n(\omega)$ when the pulse passes through the crystal or metamaterial, i.e. it deforms. This effect is called dispersion propagation. The spatial analogue of this effect is the diffraction of a beam with a finite aperture. Calculations have shown that dispersion propagation plays a significant role when the duration of pulses is 10^{-13} sec. and less. The terms dispersion propagation length and dispersion propagation time are used to evaluate the effect of dispersion propagation effect [9]:

Dispersion propagation length

$-l_d=(\tau_1^2/2)(d^2k/d\omega^2)$;

Dispersion propagation time

$-\tau_d = \left[2l\left(d^2k/d\omega^2\right)\right]^2$;

Dispersion propagation is taken into account when $\tau_1 \le \tau_d$ or τ_1; $l \ge \tau_d$ are fullfild.

Here if we accept $d^2k/d\omega^2 \approx 10^{-27}$ s^2 /cm, for $\tau_1 = 10^{-12} s.$ we get $l_d = 5m,$ and for $\tau_1 = 10^{-12} s.$ we obtain $l_d \approx 5cm.$ From above data it is clear, that dispersion propagation can be neglected for processes whose duration is measured in picoseconds. Dispersion propagation should be taken into account, if the duration of pump pulse satifies condition $\tau_1 \le 10^{-13}$ sec.

Fig.1 illustrates dependences of the signal pulse reduced energy on the unitless length of metamaterial $z/_{n/l}-$ at various values of ratios $/_{n/l}/l_v$, $/_{n/l}/l_d$ and quantity $p = \gamma^2\tau^4$. In CIA in ordinary media this dependence has a clearly maximum corresponding the coherent length [10]. However, in metamaterials, the higher values of energy are obtained not at output of metamaterial, but at its input. Such a behavoir is explained by the fact, the energy flux of signal pulse is directed opposite to the energy fluxes of both pump and idler pulses. Note that in CIA we have theoretically investigated a number of tasks involving frequency conversions in metamaterials [11-14]

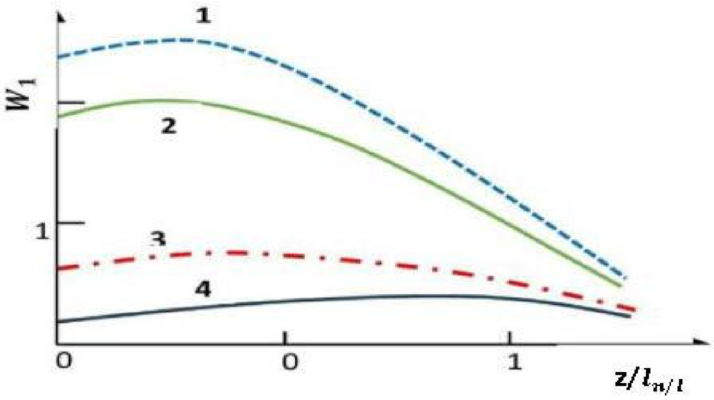

Fig. 1. Dependence of energy density of a signal wave on the relative length $z/_{n/l}$ of metamaterial at various values of ratios $/_{n/l}/l_v$, and $/_{n/l}/l_d$ $(\Delta l_{n/l} = 3)$ and $p = \gamma^2\tau^4 = 5$ (1-st, 3-rd and 4-th curves) and $p = 0$ (2-nd curve): $1-/_{n/l}/l_v = 6,$ $/_{n/l}/l_d = 0$; $2-/_{n/l}/l_v = 6$, $/_{n/l}/l_d = 0,$ $3-/_{n/l}/l_v = /_{n/l}/l_d = 3,$ $4-/_{n/l}/l_v = 0$, $/_{n/l}/l_d = 6.$

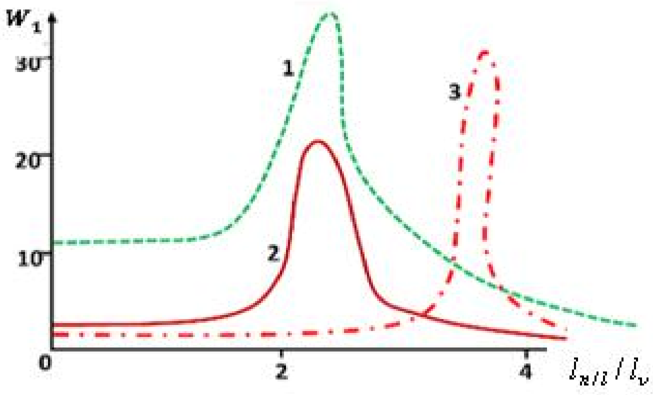

Fig. 2. Dependence of reduced energy density of a signal pulse on the ratio $l_{n/l}/l_v$ various values of the ratio $l_{n/l}/l_d$ and the parameter $p = \gamma^2\tau^4$. $(\Delta l_{n/l} = 0)$ $z/l_{n/l} = 0.5$: $1-l_{n/l}/l_d = 1$ and $p = 0$ (2-nd curve): $2-l_{n/l}/l_d = 1,$ $p = \gamma^2\tau^4 = 10$; $3-l_{n/l}/l_d = 3,$ $p = \gamma^2\tau^4 = 0$.

From the comparison of the 1-st and 2-nd curves it is seen, that the energy of signal wave decreases with inctrease in frequency modulation coefficient.

In Fig.2 the dependences of reduced energy on the ratio $/_{n/l}/l_v$ at various values of ratio $/_{n/l}/l_d$ are illustrated

As can be seen, maxima of reduced energy density displaced toward larger values of ratio $l_{n/l}/l_\nu$ with increase in the ratio $l_{n/l}/l_d$. An increase in the modulation coefficient leads to the increase in energy of the signal pulse.

The reduced energy density of a signal pulse is depicted as a function of ratio $l_{n/l}/l_d$ in the Fig.3.

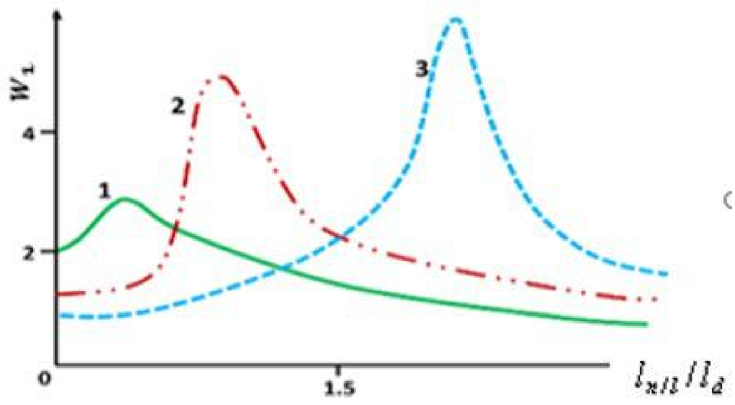

Fig. 3. Dependence of reduced energy density of a signal pulse on the ratio $l_{n/l}/l_d$ at various values of ratio $l_{n/l}/l_\nu$ and parameter $p=\gamma^2\tau^4$, $(z\Delta l_{n/l}=0.5)$, $\Delta l_{n/l}=0$ and $p=\gamma^2\tau^4=0$: $1-l_{n/l}/l_\nu=1$; $2-l_{n/l}/l_\nu=2$, $3-l_{n/l}/l_\nu=3$.

3. CONCLUSIONS

Thus one conclude, that as a result of overlapping wave fronts propagating in opposite directions, the effective interaction can occur in different parts of the metamaterial and at different time instants, which is related to the dispersion of group velocities. Dispersion of group velocities also causes deformation of the pulse shape. By adjusting the parameters of the metamaterial and the characteristics of the idler pulse, it is possible to control the overlap of the counterpropagating wave couplings and thus the frequency conversion efficiency.

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