GENERALIZED HAMILTONIAN OF THE RESONANT METRIC OF THE ACTIVE STATE OF THE MEDIUM

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A theoretical model of the Hamiltonian is proposed for spatial systems with an inhomogeneous metric of the active state of the medium, possessing internal resonant properties. The structural and geometrical description of energy, interrelated with the local metric and the excitation field, enables the consideration of excited states, including spin waves, solitons, and stable topological configurations, while accounting for the self-consistent influence of the medium and its hidden symmetry, which extends beyond the framework of traditional theoretical potentials. The representation obtained within this model proves particularly effective in the interpretation of both theoretical and experimental studies of materials with nonlinear responses and spin structures, such as ferrite films, where anisotropy and geometrically induced effects play a key role.

A model of spin rotation in a local density shell is proposed, arising in the case where the characteristic size of the shell is smaller than the characteristic radius of a magnetic domain, in which the shell is treated as a region with a continuous density gradient, realizing the rotation of the spin vector due to the internal geometry of the excited state and the medium's response.

Keywords: inhomogeneous metric, Hamiltonian, resonant properties, geometry.
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1. INTRODUCTION

Modern materials with nonlinear responses and spin structures—such as ferrite films—exhibit complex excited states whose stability is governed by the combined actions of geometry, anisotropy, and the internal structure of the medium. The variety of observed topological configurations, spin waves, and localized modes demands models that can account for both microscopic properties and the macroscopic spatial organization of the substance [1–3].

In the traditional view, the Hamiltonian of a quantum system is constructed as the sum of kinetic and potential energy, with the potential regarded as an external function of the coordinates. In the currently most popular computational approaches based on the pseudopotential method, the potential energy is defined using a set of predominantly empirical pseudopotentials, individually designed for specific elements and matched to the given crystalline structure. Such potential, as a rule, reflect the electronic and ionic properties of the material, but do not take into account the influence of the form of the excited state on the distribution of the properties of the medium itself. As a result, the calculation is based on the premise that the structure and potential are fixed, and the excitation is external.

However, in real systems — especially in materials with active media and internal anisotropy—such a separation is not always valid. The excited state and the geometry of the medium may form jointly, requiring a fundamentally different approach in which the Hamiltonian incorporates geometry, excitation, and the self-consistent response of the medium as a unified whole.

However, in experimental systems where excitation and response form a unified configuration, calculations based on fixed potentials result in a distorted description of interactions and lose predictive power. Just as in the self-consistent field of Hartree–Fock theory, where the particle's state determines the potential landscape, in our model, the geometry of the medium itself becomes a self-consistent parameter evolving with the excitation. This creates the need for a model in which energy interaction is determined not only by the chemical nature of the elements but also by their position within a spatially varying metric. This means that the shape, the medium's response, and the structure of the excited state are inseparably linked, and geometry becomes an active element of the interaction. Notably, even in the Helmholtz wave equation, a close connection between the shape of the medium and localized modes emerges. The wave equation in inhomogeneous geometry reflects not merely the energy distribution but also lays the foundation for the stability conditions of states emerging in the resonant structure of the medium. This is observed in acoustics, optics, and spin systems, where the eigenmodes reflect not only the frequency spectrum but also the spatial configuration of the material. Thus, the geometry of the medium may play not a passive, but an active role in the formation of excited states.

The problem of describing an effective potential in a medium undergoing internal structural and symmetry transformations has been discussed in the context of works that laid the foundation for the concept of pseudopotential. However, historically, the dominant strategy has been based on local approximations oriented toward electronic structure under a fixed metric. It is conceivable that if, at a certain point, the interpretation of X-ray spectra had been extended to include geometric context — considering possible Riemannian distortions of density — our understanding of potentials and excited states might have developed differently. It is worth noting that most spectroscopic

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methods — including X-ray diffraction, small-angle scattering (SAXS), neutron interference, and electron spectroscopy — interpret data not in direct, but in so-called reciprocal space [4-8]. This fact, having become routine, is rarely reconsidered: the spatial structure in which these spectra are constructed is, in essence, Riemannian; the spatial metric depends on density, and data interpretation requires accounting for this curvature.

If all physical consequences of excitations manifest and are interpreted in a space topologically different from the real one, should we not recognize that this space is not merely a convenient abstraction, but an active participant in interactions?

Thus, a geometric approach [6] to the description of excited states based on a Riemannian metric gains not only theoretical justification, but also instrumental relevance. It naturally aligns with the practice of interpreting experimental data and can be used to model medium properties in terms of conjugated geometry.

In this work, we propose a Hamiltonian model that incorporates the spatially variable metric of the medium, its internal response, and the exciting field. This formulation allows for a self-consistent account of the mutual influence between the medium and excitation and provides a framework for describing stable excited states and topological configurations while taking geometry into account. The theoretical basis of the model lies in the concept of conjugated geometry, where the medium's metric not only affects the distribution of densities but also determines the structure of the interaction itself. Therefore, the goal of the present work is formulated in terms of a theoretical model describing excited states in systems with spatially variable metric and intrinsic resonance mechanisms. The proposed Hamiltonian accounts for the local geometry, the exciting field, and the active response of the medium, offering a more comprehensive description of stable states, topological configurations, and phase transitions in complex materials, such as ferrite films with pronounced spin structure.

2. THE HAMILTONIAN OF AN ACTIVE METRIC

The Hamiltonian is not merely a mathematical approximation, but rather an expression of the internal structure of a quantum system and its evolution, observed through the Schrödinger equation. In every quantum system, there exists a specific response to internal stresses and external perturbations. It is this response, born from the medium itself, that we refer to as the Hamiltonian.

Traditionally, we mathematically distinguish two main components of the Hamiltonian: the kinetic and the potential terms. The former is associated with fast, relatively light particles and reflects their spatial dynamics; the latter—with slow, heavy components of the medium, describing configurations, interactions, and vibrational degrees of freedom. This separation is convenient for analysis but does not always correspond to the true interconnected nature of a quantum system. In particular, in systems with a geometrically active metric and excitable medium, this division loses clarity, since motion and interaction become interpenetrating aspects of a single response.

When solving the stationary Schrödinger equation, the eigenstates of the Hamiltonian form a complete orthonormal basis of Hilbert space—a infinite-dimensional generalization of Euclidean space, complete under the metric induced by the scalar product. However, the structure of the Hilbert space depends on the geometry in which the Hamiltonian operator is defined. The classical formulation of the Schrödinger equation assumes that the Hamiltonian is constructed on a fixed geometry—typically on a flat Euclidean metric. In such geometry, the Laplacian (in the kinetic term) is defined as the sum of second derivatives with respect to Cartesian coordinates, and the potential energy—as an external function of position. But in physically saturated systems, where excited states, density fluctuations, local anisotropies, and medium inhomogeneities are present, the assumption of fixed geometry becomes too restrictive. The behavior and stability of the wavefunction often depend on the specific space it "lives" in. This suggests the need to revise the fundamental object—the Hamiltonian itself—toward its geometric dependence. In this context, the idea of geometrization emerges transitioning from a fixed, a priori metric to a spatially variable one, generated by the excited state itself.

A localized excited state of the system can be described by the energy density ρ(x). This quantity, in turn, can induce its own geometry, altering the spatial metric. Consequently, the metric becomes not an external parameter, but a function of the density field:

$$g_{ij}(x) = \delta_{ij} + \alpha \frac{\partial_i \rho(x) \partial_j \rho(x)}{\rho(x)^2 + \varepsilon} \qquad (1)$$

where α is the scale sensitivity coefficient, and ε is a regularize that smooths behavior near vanishing density. This form of the metric describes local curvature of space induced by the internal properties of the medium itself and replaces external geometry with an effective, self-consistent one.

After this transformation, the Hamiltonian is naturally modified. The kinetic part is replaced by the Beltrami Laplacian defined on a Riemannian manifold with the metric $g_{ij}(x)$ [9-12]:

$$\widehat{H}_g = -\frac{\hbar^2}{2m\sqrt{|g(x)|}}\, \partial_i \sqrt{|g(x)|}\, g_{ij}(x) \partial_j + V(x, g) \qquad (2)$$

where $\sqrt{|g(x)|}$ is the square root of the determinant of the metric tensor, defining the volume density of the measure.

It is evident that the operator now depends not only on the coordinates, but also on the derivatives of the density, while the potential energy *V(x,g)* acquires a geometric meaning: for example, it may include the

scalar curvature *R(x)*, characterizing the local curvature of space. Thus, even with a fixed form of interactions, potential energy becomes a function of the medium and its geometric organization.

The normalization of the wavefunction also changes. In contrast to the standard measure d^nx, the normalization integral is now taken over the volume in geometrically curved space:

$$\int |\psi(x)|^2 \sqrt{|g(x)|}\, d^n x = 1 \qquad (3)$$

Note that this leads to an important consequence: localization and probability distribution themselves obey the geometry induced by the excitation. Not only does the wave adapt to the medium, but the medium (through the metric) "draws" the wave into its own curvature. A sharp question arises here: if no external geometry is specified, how can the emergence of curvature be explained? Note that curvature is not introduced externally and is not imposed as a structure of space. It is constructed as a derivative of the excitation density; that is, the radius of curvature is not a geometric quantity but a function of the density gradient and its derivatives. In this way, geometry is generated by matter, not by a background. Thus, the transition from fixed to self-consistent metric opens the possibility to describe localization, stability, and topological protection within a single equation. The Hamiltonian transforms from an operator on a flat background into a dynamic structure reflecting the internal organization of the medium, thereby providing a more accurate description of coherent states, nonlocal interactions, and self-organizing modes, especially in complex materials where traditional flat geometry loses its adequacy.

The following form of the Hamiltonian is proposed, in which the metric creates spatial modulation of the internal properties of the medium. It is not a fixed geometry but depends, for example, on the distribution of interaction forces such as anisotropic exchange or defects:

$$\hat{H} = T\left(\phi(\vec{r}),\ g_{ij}(\vec{r})\right) + V_{res}\left(\phi(\vec{r}), \phi^{(0)}, g_{ij}(\vec{r})\right) \qquad (4)$$

where $\phi(\vec{r})$ is the form of the state (e.g., a wavefunction or spin field); $g_{ij}(\vec{r})$ is the local metric tensor of the medium, reflecting anisotropies and nonlinear structure; $\phi^{(0)}$ is the reference (background or unexcited) state; and V_{res} is the resonance potential reflecting the self-consistent connection between the form and the geometry of the medium.

It should be emphasized that the exciting influence ∇S acts as an operator deformation of the Hamiltonian as a whole. In the generalized approach, the exciting effect on the system is interpreted as the action of an operator $\hat{S}$, representing the gradient of the external field, on the Hamiltonian of the system. This action leads to a change in both the kinetic and potential parts and can be expressed through the commutator:

$$\delta\hat{H} = \left[\hat{S}, \hat{H}\right] + \cdots \qquad (5)$$

This formulation reflects that the Hamiltonian is not a static functional but undergoes self-consistent deformation under the influence of the exciting field. Particularly important is that, in the case of a geometrically active metric, this action affects the structure of $g_{ij}(x)$, thereby altering both the kinetic and potential parts not separately but jointly — through variation of the state form φ and the density.

Let us describe the action of the exciting field on the system through the variation of the Hamiltonian:

$$\delta\hat{H} = \left[\hat{S}, \hat{H}\right] = \delta T + \delta V_{res} \qquad (6)$$

It becomes clear that this formulation allows one to consistently consider both resonance changes in the kinetic term and local restructuring of the potential — i.e., the exciting field influences not just a specific term of the Hamiltonian, but the structure of its action.

Thus, within the generalized model, the Hamiltonian is not a fixed sum of kinetic and potential energy but is formed as the result of self-consistent dynamics of the medium in the presence of an exciting influence. Each part of both T_{res} and V_{res} — depends on the shape of the state ϕ, the metric g_{ij}, and on the operator action ∇S, which reflects the effect of external excitation. The resulting non-local and dynamically adaptive behavior can be represented in the form of an additional excitation functional *W*, which includes the density response, time derivatives, and field structure:

$$\hat{H}_{total} = T_{res}\left(\phi(\vec{r}),\ g_{ij}(\vec{r})\right) + V_{res}\left(\phi(\vec{r}), \phi^{(0)}, g_{ij}(\vec{r})\right) + \hat{W}[\delta\rho, \partial_t\rho, \phi(\vec{r})] \qquad (7)$$

where $\hat{W}$ is not an external addition but arises because of the deformation $\left[\hat{S}, \hat{H}\right]$ under medium activity conditions; T_{res} is the resonant form of the kinetic term. Note that the absence of background consideration in the kinetic term raises a quite specific question: “relative to what is it defined?” A way out is possible if we introduce the background gradient, for example in the form:

$$\breve{\phi}(\vec{r}) = \phi(\vec{r}) - \phi^{(0)}(\vec{r}) \qquad (8)$$

term takes the form $T_{res}\left(\phi(\vec{r}), \phi^{(0)}, g_{ij}(\vec{r})\right)$, which significantly changes the physical interpretation since we can observe deviations from the metastable state that defines the “calm” configuration of the medium. This approach emphasizes the role of the background as a dynamic state dependent on the metric and opens the

possibility of studying the conditions of its stability, violations, and phase transitions. Thus, equation (8) allows us to describe both stationary and excited states while considering the spectral, geometric, and dynamic relationship between form and medium. Note that the resonant form of the kinetic term has the form:

$$T_{res} = \int \gamma(\vec{r})\, g_{ij}(\vec{r}) \partial_i \varphi^* \partial_j \varphi \cdot R_{dyn}(\varphi, \nabla S, \nabla \varphi) \sqrt{|g|}\; d^n x \tag{9}$$

Here, R_{dyn}n describes the coordination of local dynamics of the form with the exciting influence, creating a mechanism of self-organization, as well as enabling the description of observed spectral features not explainable within the framework of static medium properties. A typical example of when the resonant structure manifests in the kinetic term of the Hamiltonian is electron–phonon interaction. In such systems, the motion of the electron excites the phonon field, and the kinetic energy ceases to be independent and becomes a function of the medium's response. In the case of the Fröhlich model [13], the electron momentum operator enters the effective Hamiltonian on equal footing with the phonon creation and annihilation operators. This mechanism shows how excitation dynamics can resonate with the form of the medium and create non-local collective effects. Note that the potential functional is also not imposed externally but is formed as an internal projection of consistency between the form of the state and the medium's response. Its structure can be represented as a sum of three functional contributions: f_{res} — resonance interaction between the exciting field and the form; f_{sym} — consistency of the form with local symmetry; f_{top} — topological characteristics (e.g., vortices, domain boundaries). This approach allows one to build a potential sensitive to the local phase structure and capable of reflecting the complex multilayered nature of excited states in materials [3]:

$$V_{res} = f_{res}\,(\phi(\vec{r}), \nabla S) + f_{sym}(\phi(\vec{r}), G_{env}) + f_{top}\big(\phi(\vec{r})\big) \tag{10}$$

A more detailed analysis of this expression indicates that although the three functional contributions to the potential energy differ in origin, they all depend to some extent on the exciting influence transmitted via operator S. The resonant f_{res} directly includes ∇S as an argument, defining the local response of the form to the external field. The symmetry contribution f_{sym} changes with the breaking or restoration of local symmetry of the medium caused by excitation. The topological functional f_{top}, although carrying information about the global configuration, is also sensitive to the action of the exciting field, as the birth, movement, or restructuring of topological objects — including domain walls, vortices, and soliton formations — may occur. Thus, ∇S participates not only in shaping the dynamics but also in redefining the energetic structure of the state, as well as its stability under the given excitation.

3. VARIATIONAL FORMULATION AND EQUATION OF MOTION

The generalized structure of the Hamiltonian describes the possible energy of the system but does not determine which configuration of the form ϕ is realized in each medium. For this, a principle is needed that connects the form with the minimum energy, that is, a variational approach. It is precisely this that allows us to define the admissible states as those in which the form and the response of the medium agree, expressed through the stationarity of the energy functional.

We employ the Lagrange equation — one of the main tools of variational calculus. It is widely used in optimization problems and, together with the principle of stationary action, is used to compute trajectories in mechanics. The use of the Lagrange equation to find the extremum of a functional is in some sense analogous to the classical differential calculus theorem stating that a smooth function can have an extremum only at a point where its first derivative vanishes. In the case of a function of a vector argument, this means the vanishing of its gradient.

In our case, the Lagrange equation is applied to the energy functional to derive a generalized equation of motion:

$$\delta E[\phi] = 0 \tag{11}$$

It is worth noting that in quantum mechanics, according to the Copenhagen interpretation, it is not required to know the exact way in which a particle moves. The particle moves from the initial state to the final state along all imaginable paths simultaneously (which, obviously, are infinite in number). The probability amplitude of the transition from one given state to another is the sum of amplitudes along all these paths. Thus, by taking equation (11) into account, we obtain:

$$\widehat{H}(\phi) = \int_\Omega \left[\frac{\hbar^2}{2m} g_{ij}(\vec{r}) \partial_i \phi^* \partial_j \phi + V_{res}\left(\phi(\vec{r}), \phi^{(0)}, g_{ij}(\vec{r}), \nabla S(\vec{r})\right)\right] \sqrt{|g(\vec{r})|}\; d^3 x \tag{12}$$

$$\delta E[\phi] = \delta \left\{ \int_\Omega \left[\frac{\hbar^2}{2m} g_{ij}(\vec{r}) \partial_i \phi^* \partial_j \phi + V_{res}\left(\phi(\vec{r}), \phi^{(0)}, g_{ij}(\vec{r}), \nabla S(\vec{r})\right)\right] \sqrt{|g(\vec{r})|}\; d^3 x \right\} = 0 \tag{13}$$

from which the following form of the generalized equation of motion follows:

$$-\frac{\hbar^2}{2m}\nabla_i\left(g_{ij}\nabla_j\phi\right)+\frac{\delta V_{res}}{\delta\phi^*}=0 \qquad (14)$$

In the first term, the covariant derivative operator accounts for the geometry of the medium via the metric, while the second term describes the functional derivative of the potential with respect to the form. This equation describes the evolution of the form in space, where the medium possesses an active metric and its own spectrum of responses, allowing the determination of both stable and transitional states.

It is interesting to note that under conditions where the form interacts with a topologically active medium (for example, in the presence of domain structures or vortex shells), the realization of a Möbius geometry is possible — a configuration with a single side, analogous to Riemann surfaces. Such a Möbius-like structure naturally describes the spin rotation and its change in orientation during the continuous development of the form. In this context, even the classical equation of motion allows for an expanded interpretation: a spin texture may take on values corresponding to traversal along a Möbius surface, reflecting the complex topological connectivity of the medium and the form.

Of particular interest is the behavior of spin in the shell of domain structures, where continuous rotation occurs within the layer. If the spin completes a full turn along a closed path but returns in the opposite direction — an analogue of a Möbius strip structure arises. Such behavior captures not only the topological irreducibility of the transition but also the orientational bilayeredness of the medium, where the spin "passes through itself" as if walking along a single but twisted surface.

Formally, such a structure is described by an embedded Riemann surface with non-constant orientation, allowing smooth transition between η=+1 and η=−1. This does not require modification of the equation of motion but emphasizes that the solutions may include trajectories analogous to Möbius cycles and thus possess a dual nature of response.

Thus, even without changing the analytical form of equation (14), the internal geometry and topology of configurations become part of the form's dynamics — especially in the case of resonant or excited states.

4 EXAMPLES

4.1 LOCALIZED EXCITATION IN ANISOTROPIC METRIC

Let us consider a specific case: a localized excitation $\phi(\vec{r})$ propagating in a ferrite film with an anisotropic metric that depends on the distribution of Zn ions and oxygen n defects. Suppose the metric has the form:

$$g_{ij}(\vec{r})=\delta_{ij}+\alpha_{ij}f(\vec{r}) \qquad (15)$$

where α_{ij} is the tensor of local anisotropy, and $f(\vec{r})$ is the distribution function of impurities, obtained, for example, from EPR experiments or microscopy. In this case, the background $\phi^{(0)}$corresponds to a stationary configuration of the spin distribution obtained at $S(\vec{r})=0$, and the exciting field models the action of an external magnetic pulse:

$$\nabla S(\vec{r})=Aexp\left(-\frac{|\vec{r}-\vec{r}_0|^2}{\sigma^2}\right)\hat{n} \qquad (16)$$

where *A* is the amplitude $\vec{r}_0$ is the center of the pulse; σ- is the distribution width; $\hat{n}$ is the direction of the impact. The functional V_{res} in this case can be constructed as:

$$V_{res}=\lambda_1\left|\nabla S\cdot\left(\phi-\phi^{(0)}\right)\right|^2+\ \lambda_2 Tr\left[g_{ij}^{-1}\partial_i\phi^*\partial_j\phi\right]+\ \lambda_3 Q(\phi) \qquad (17)$$

where $Q(\phi)$ is the topological contribution, for example, the vortex density or "winding number," evaluated by the expression:

$$Q(\varphi)=\frac{1}{4\pi}\int\varphi\cdot\left(\partial_x\varphi\times\partial_y\varphi\right)d^2r \qquad (18)$$

Here, ϕ is interpreted not as an amplitude but as a vector or spin field, with a phase rotating by 2π or more along the shell. In general, the local spin rotation can be described as:

$$\vec{n}(\vec{r})=\frac{\varphi(\vec{r})}{|\varphi(\vec{r})|^2};\qquad \omega(\vec{r})=\vec{n}(\vec{r})\cdot\left(\nabla\times\vec{n}(\vec{r})\right) \qquad (19)$$

where $\omega(\vec{r})$ is the local frequency (density) of spin rotation. Naturally, this leads to the introduction of a topological number of the shell, describing how many times the spin turns along a closed path:

$$Q(\vec{n})=\frac{1}{4\pi}\int\vec{n}\cdot\left(\partial_x\vec{n}\times\partial_y\vec{n}\right)d^2r \qquad (20)$$

This is the *skyrmion number*, indicating topological protection of the configuration. Thus, the domain shell can be interpreted as a phase boundary with dynamic modulation, inside which

continuous rotation occurs, local decoherence or resonance is possible, and the nature of excitation changes (vector rotation instead of scalar amplitude). Since the spin reverses orientation along a closed path, a topological Möbius strip–type structure arises. Such configurations cannot be formed through local deformations and play a key role in the stability of magnetic states, determining both the spectrum of oscillations and possible transitions between states.

4.2 SPIN STRUCTURE IN A MEDIUM WITH CHIRAL METRIC

Let us consider a $Ni_{1-x}Zn_xFe_2O_4$ ferrite film, where chirality of the structure arises due to the local symmetry breaking in the distribution of Zn over tetrahedral positions. Such a distribution leads to inhomogeneous exchange interactions and the formation of an effective chiral metric (note that a chiral metric, in the general sense, refers to the property of an object that cannot be superimposed onto its mirror image). In this case, the form describes a spin configuration (for example, of the skyrmion type), and the excitation field simulates an external magnetic field or a temperature gradient [15-28]. The geometry of the medium is defined by a tensor representing local anisotropies of exchange and magneto dipole interactions. For constructing the functional, published data may be used, such as exchange interactions, the anisotropy tensor, as well as local defects in Zn distribution, forming the metric. This allows linking the geometry of the medium with stable magnetic configurations and predicting behavior under external influences.

Thus, in our understanding, the form ϕ describes the spin configuration (e.g., skyrmion type), the excitation field ∇S simulates the external magnetic field or temperature gradient, and the geometry of the medium is defined by the tensor $g_{ij}(\vec{r})$, representing local anisotropies of exchange and magnetodipole interactions. The metric changes during local phase transitions and can be reconstructed from experimental data on the Zn distribution. Obviously, for the considered system, the potential functional will include contributions from chiral symmetry, interaction with the external gradient, and stable vortex structures emerging at the boundaries of Zn-ordering regions. This approach allows one to relate the geometry of the medium to stable magnetic configurations and predict the material's behavior under changing external conditions. The evolution equation of the form ϕ and the metric g_{ij} takes the dynamic form:

$$\partial_t \phi = -\delta H/\delta\phi; \quad \partial_t g_{ij} = F[g_{ij}, \nabla\phi, \nabla S] \tag{21}$$

where F is a function depending on the gradients of the form and the excitation field. Thus, the model describes the joint evolution of the spin configuration and the geometry of the medium, consistent with the experiment and reflecting observed phase and topological transitions.

To evaluate the stability of the spin configuration $\phi(\vec{r})$ in the chiral metric $g_{ij}(\vec{r})$, consider variations of the form $\delta\phi(\vec{r})$ and study the oscillation spectrum near a stable solution. In the linear approximation, the variation of the energy function leads to a perturbation operator:

$$\delta^2 E(\phi) \approx \int \delta\phi^* \left[-\frac{\hbar^2}{2m} \nabla_i (g_{ij} \nabla_j) + \frac{\delta^2 V_{res}}{\delta\phi \delta\phi^*} \right] \delta\phi \sqrt{|g(\vec{r})|}\, d^3x \tag{22}$$

The chiral metric affects the eigenvalues of the operator (excitation spectrum), creating asymmetry between left and right rotations in spin structures. This leads to characteristic shifts in magnon modes and, consequently, to the effect of "magnon Hall effect", especially pronounced in ferrites with local Zn inversion. Assuming that the main form ϕ has skyrmion topology and, therefore, the metric has spiral modulation $g_{ij} \sim \delta_{ij} + \epsilon_{ijk}\chi_k(\vec{r})$, where $\chi_k(\vec{r})$ is the tensor of chiral modulation generating spiral distortion in space associated with the breaking of "mirror" symmetry, not only does stability of the configuration arise, but also local energy barriers protecting the vortex structure from decay. Such topological barriers are the result of mutual "matching" of the form ϕ, the gradient of the excitation field ∇S, and the metric tensor g.

To describe the evolution of the state form $\phi(\vec{r}, t)$ in a medium with a chiral metric, the variational form of the motion equation must consider the form of the resonance potential (10), that is:

$$i\hbar \frac{\partial \phi}{\partial t} = -\frac{\hbar^2}{2m} \nabla_i \left(g_{ij}(\vec{r}) \nabla_j \phi \right) + f_{res} + f_{sym} + f_{top} \tag{23}$$

in which each functional term accounts for the local distribution and excitation conditions. This equation describes the dynamics of the spin configuration in a medium with local anisotropy modulated by chiral distortions. It allows modeling not only stable states (e.g., skyrmions) but also transitional processes between them under external influences.

Figure 1 visualizes the state form $\phi(\vec{r})$ in a chiral metric: distortion introduced by asymmetric contributions to the metric $g_{ij}(\vec{r})$ leads to modulation of amplitude and phase patterns. The image reflects local vortex and front-like structures capable of forming stable states (e.g., skyrmions). Figure 2 visualizes the metric on the complex plane, which is directly related to Riemannian geometry. Such visualization gives an idea of how local space distortions (in our case, the metric $g_{ij}(\vec{r}, t)$)can be represented as conformal transformations or deformations of the plane, especially

when superimposed on the dynamic field $\phi(\vec{r},t)$. Note that in our model, the physical field ϕ is modeled against a background deformed by the active medium. In the Riemannian metric, it is expressed through a local scaling factor and reflects the metric, its geometry, distortions, and local features. Figure 3 shows a distorted image of the Riemannian metric. The vector field shows how each element of the plane is distorted under the action of chiral transformation — the background does not remain unchanged; everything depends on the local state. This is not just a pretty picture. It shows how the chiral metric alters trajectories, symmetry, and local phases; each arrow is a possible trajectory of spin response, depending on the metric $g_{ij}(\vec{r})$. In the center, a skyrmion or soliton may form — a stable configuration emerging from the concordance of form ϕ and metric. Figure 4 shows the distribution of the density $|\phi(\vec{r})|^2$along with the aforementioned field. This provides a complete picture of the interaction between form and geometry. One can clearly see how the metric structure induces local excitation

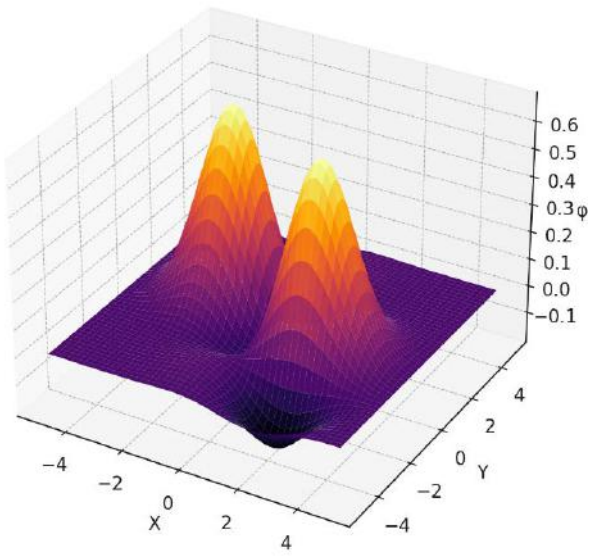

Fig. 1. Configuration of the spin density $\phi(\vec{r})$ for the chiralmetric

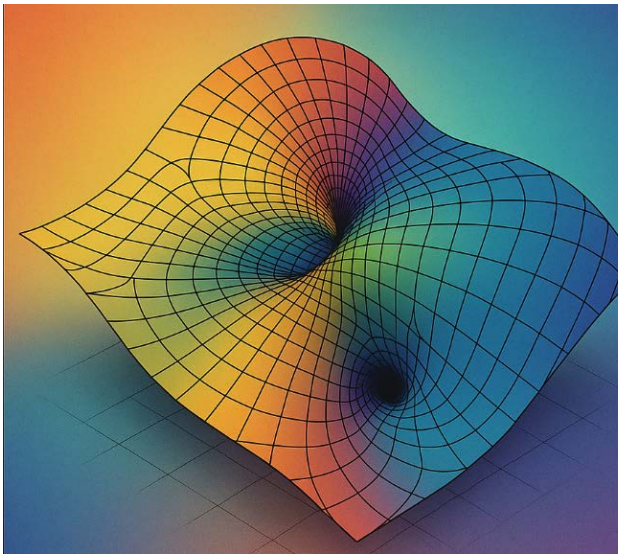

Fig. 2. Metric on the complex plane (Riemannian projection).

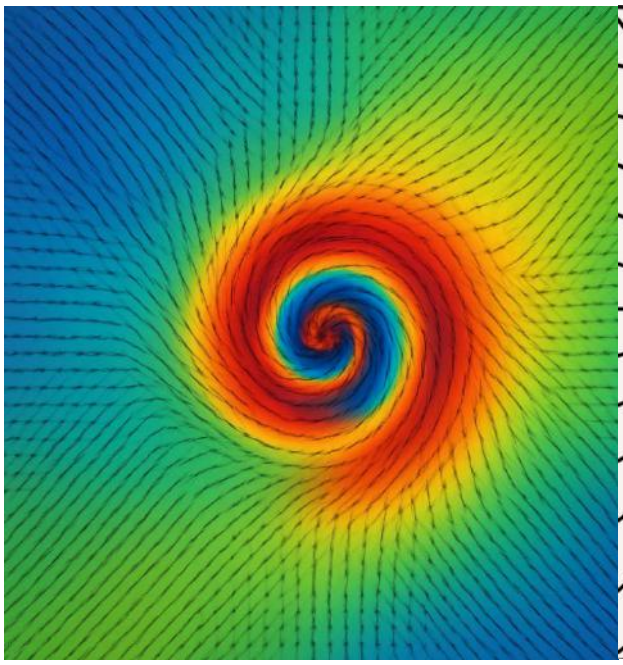

Fig. 3. Vector field of chiral deformation.

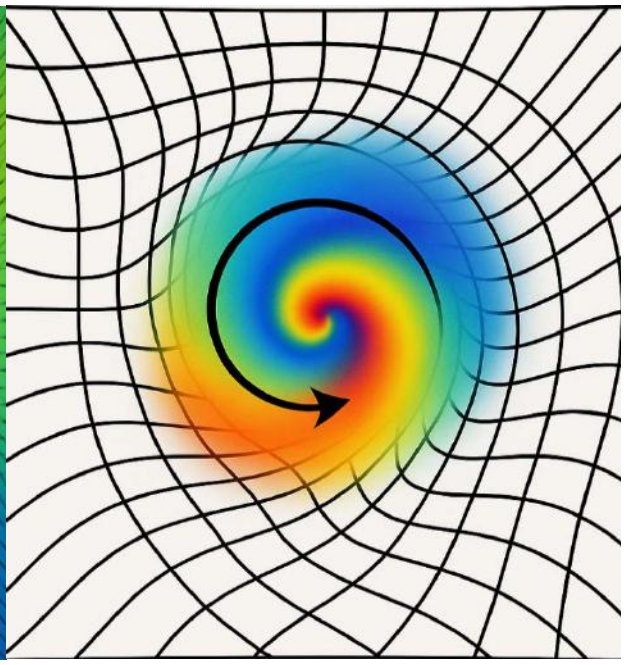

Fig. 4. The modulus field $||\phi(\vec{r})|^2$ imposed on the metric.

4.3 ON SPIN ROTATION IN THE SHELLS OF MAGNETIC DOMAINS IN $Ni_{1-x}Zn_xFe_2O_4$ FERRITES

In ferrites of the $Ni_{1-x}Zn_xFe_2O_4$ type, a magnetic domain is a region with a uniform spin orientation [24]. However, the key element is the domain shell — a transition zone where coordination between the internal structure and the surrounding medium occurs. In the context of the model based on a spatially variable metric and the medium's participation in the excitation process, the shell is a structure composed of nested Riemannian surfaces that sequentially change their geometry. Each of these surfaces performs a metric shift, thus providing a transition from the metric of the internal domain region to that of the external environment. A 180° spin rotation in this case should not be interpreted as a physical rotation of the vector $\vec{S}$, but rather as a geometrically induced change of the coordinate frame in which this spin is fixed. Thus, the spin retains its orientation in the local coordinates of the shell, but due to the gradual transformation of the surface geometry, we observe an external change in direction. This is the key difference of our model: the spin does not rotate — it is the space in which it is anchored that rotates. Since the medium is described by its own geometry (in the limit — by its own energy levels and density distribution), the matching of excitation and medium occurs in the interaction space between them. This is not just a boundary between two states, but a functional geometric interface ensuring the transmission of orientation, momentum, and energy.

In mathematical terms, this process can be represented as a sequence of local, successive metric rotations. Each such rotation is associated with a micro-change in the geometry of the medium and the excited state. The sum of these coordinated shifts leads to a macro-effect — the rotation of orientation. In the limiting case, this is expressed as:

$$\sum_i \delta\theta_i = \pi \tag{24}$$

which corresponds to full coordination of direction across an interval between two states with opposite orientations — the sum of local rotation angles within the shell leads to a complete orientation change.

It is important to note that the "Riemannian" shell has a discretely analog nature: it allows nested transitions but is formed sequentially, with a limit on the speed of coordination propagation. That is, a finite number of Riemann surfaces clearly exists, each of which can be associated with a node of the coordinating layer. Consequently, the shell is not just a "boundary" of some cluster, but a dynamic structure actively responding to excitation. And it is this dynamic process, unfolding in the space between two metrics — of the domain and the medium — that leads to the observable, seemingly physical spin vector rotation when projected into Euclidean space.

Thus, spin and polarization effects in $Ni_{1-x}Zn_xFe_2O_4$-type ferrites acquire a new interpretation: as the result of a geometrically induced process of state transmission, realized in a "Riemannian" shell, which casts doubt on assumptions about the existence of so-called mirror-reflection excitation structures in the medium. Everything takes place in the interaction space — the only reality that leaves a quite observable trace. The reason for the 180-degree spin rotation is the energetic coordination of two states with opposite spin orientations, in which the system "finds" the minimum energy trajectory between them.

Consequently, if inside the domain the spin is, say, directed upward — $\vec{S}_{\uparrow}$, and outside the domain downward -$S_{\downarrow}$, a continuous rather than abrupt rotation occurs:

$$\vec{S}(x) = S_0(\sin\vartheta(x), 0, \cos\vartheta(x)),\ \vartheta(x) \in [0, \pi] \tag{25}$$

To properly understand the properties of the Riemann surface, note that it is a local region of a two-dimensional surface on which a complex structure and metric are imposed. This is an isotropic conformal distortion of the plane, defining the geometry of the spin shell, which is described as a two-dimensional Riemannian surface with a scaling factor λ(x,y) reflecting the deformations:

$$ds^2 = \lambda(x,y)^2(dx^2 + dy^2) \tag{26}$$

where λ(x,y) is the scaling factor indicating how stretched or compressed the metric is at each point. As a result, even if the surface appears flat, it is geometrically distorted. And how would a spin "lie" on it? It cannot be just a vector: either we observe vector stratification (for example, a spinor field), or a vector on the tangent plane changing direction under parallel transport. In Riemannian space, parallel transport depends on curvature (this is the law of parallel transport in Riemannian space, where the vector's rotation is determined by the curvature of the path):

$$\delta S^i = -\Gamma^i_{jk} S^j dx^k \tag{27}$$

where Γ^i_{jk} – are the Christoffel symbols. If the trajectory is closed, a holonomy angle arises (in differential geometry, holonomy refers to the parallel transport of vectors around a closed loop in curved space), which results in the vector returning to its original location rotated. This geometric phase — akin to the Berry phase in quantum mechanics — reflects the intrinsic curvature of the space, rather than an external force or torque.

In our case, this means that the spin vector, being transported along the Riemannian shell, undergoes an effective rotation purely due to the shell's geometry. The observed 180° rotation is not a local event, but the result of an integrated geometric transformation. The shell thus acts as a mediator, transforming the internal spin orientation to match the external one through continuous, metric-driven alignment.

This geometric mechanism explains the robust stability of domain configurations and the difficulty of disrupting such spin structures by local perturbations. Any transition must overcome the entire geometric path encoded in the shell. It also suggests the existence of quantized excitations — topologically protected modes — that can propagate along the shell or between domains without being scattered by imperfections in the crystal lattice.

This conceptual shift — from dynamics in fixed space to dynamics of space itself — redefines our understanding of magnetic systems. Instead of treating geometry as a passive backdrop, it becomes an active participant, guiding and constraining physical processes. The domain shell becomes a dynamic Riemannian interface, encoding topological information, facilitating energy minimization, and enabling new modes of excitation that were inaccessible in Euclidean approximations.

That is, if the trajectory is curved, the spin direction rotates as well:

$$\vartheta \sim \int_{\Sigma} R_{ijkl} dx^i \wedge dx^j \tag{28}$$

where Σ is the area enclosed by the closed contour, R_{ijkl} is the curvature tensor, and the integral is taken over the closed region Σ. The spin rotation is thus determined by the integral of curvature over the enclosed area — this is the formula for local holonomy and parallel transport in Riemannian space. If the field possesses spin structure, it always depends on geometry — it undergoes spontaneous rotation even in the absence of external fields. In a Riemannian medium, spin rotates because a vector cannot remain unchanged when moving along a curved path. This is a "built-in" law of rotation encoded in the geometry of space itself.

Summarizing the above, the geometric formalization of the shell via metric stratification can be expressed as follows: let $\mathcal{M}_1$ be a Riemannian surface describing the metric inside the domain, $g_{ij}^{(1)}(x)$ and $\mathcal{M}_2$ the Riemannian surface of the surrounding medium, $g_{ij}^{(2)}(x)$. The shell is defined as the region $\Omega_{shell} \subset \mathcal{M}$, where $\Delta g_{ij}(x) = g_{ij}^{(1)}(x) - g_{ij}^{(2)}(x) \neq 0$ and simultaneously the density mismatch is observed: $\Delta\rho(x) = \rho^{(1)}(x) - \rho^{(2)}(x) \neq 0$. We define the shell functional as $\mathcal{F}_{shell}[\phi] = \int_{\Omega} \left(\alpha \left\|\Delta g_{ij}\right\|^2 + \beta|\Delta\rho|^2\right)\sqrt{|g|}\, d^n x$. The minimum of this functional defines the region of maximum gradient matching — the domain shell. In this shell, $\nabla\vec{S} \neq 0$, meaning that an energy contribution arises from the gradient of the spin structure. The spin vector $\vec{S}$ describes an arc on the spin sphere from the north to the south pole — a 180° rotation. Assuming the excited metric takes the form $\tilde{g}_{ij}(x) = g_{ij}^{(0)}(x) + \delta g_{ij}(x)$, where $g_{ij}^{(0)}$- is the initial metric and $\delta g_{ij}(x)$ – arises from the interaction between the excitation and the unfilled states of the medium, we emphasize that this is not a simple linear combination, but rather a result of the coordination of local densities, the medium's response, and the excitation as conveyed through geometry. Suppose the excitation creates a localized density distribution, thus forming some energy state $E^*(x)$, which determines the new local behavior of the medium. Then:

$$\tilde{g}_{ij}(x) = f\left(g_{ij}^{(0)}(x),\ \rho_{occ}(x), \rho_{empty}(x), E^*(x)\right) \tag{29}$$

Here, $f\left(g_{ij}^{(0)}(x),\ \rho_{occ}(x), \rho_{empty}(x), E^*(x)\right)$ –is a nonlinear mapping that depends on the medium's response — a functional describing the restructuring of geometry under the energy pressure of the excited state, with $\rho_{empty}(x)$ being the generalized density of unoccupied states (above the Fermi level), which determines the energy sensitivity of the medium to excitation:

$$\rho_{empty}(x) = \sum_{E>E_F} \delta\big(E - E_n(x)\big) \tag{30}$$

The main functional of the shell can now be expressed in terms of the excited metric as

$$\mathcal{F}_{shell}[\tilde{g}] = \int_{\Omega} \left(\alpha \left\|\Delta g_{ij}\right\|^2 + \beta\delta E(x)^2\right)\sqrt{|\tilde{g}|}\, d^n x \tag{31}$$

where $\delta E(x) = E^*(x) - E_F$. From this, the metric equation can be written as:

$$\tilde{g}_{ij}(x) = g_{ij}^{(0)}(x) + \mathcal{R}_{ij}(x) \tag{32}$$

where the nonlinear response functional arising from resonant matching (of energy levels, densities, geometry, etc.) is given by:

$$\mathcal{R}_{ij}(x) = \Phi_{ij}\big(\rho_{empty}(x), E_F,\ \phi_{exc}, \hat{T}_{loc.sym.}\big) \tag{33}$$

Thus, $\tilde{g}$ can be interpreted as the total resonant response formed from the spatial structure of the excitation $\phi_{exc}(x)$, local gradients of the density of states, and the geometry of the medium itself. Therefore, the process of successive transitions between intermediate metrics $\tilde{g}_{ij}(x,\lambda)$, from the metric describing the internal domain structure g_{ij}^{dom} to the metric of the external environment g_{ij}^{env}, can be described as:

$$\tilde{g}_{ij}(x,\lambda) = \mathcal{T}_{ij}(x,\lambda) = (1-\lambda)g_{ij}^{dom}(x) + \lambda g_{ij}^{env}(x) + \mathcal{R}_{ij}(x) \tag{34}$$

in which, obviously, λ is defined in the 0≤λ≤1 interval and inside the interval the system builds its trajectory in the space of metrics, which may not be linear, but may loop, return (if we consider the Mobius and torus topologies). Thus, a structure of the type of a chain of metrics is assumed

$$g_{ij}^{(0)} \to g_{ij}^{(1)} \to g_{ij}^{(2)} \to \cdots \to g_{ij}^{(n)}$$

in which each subsequent metric can be defined as

$$g_{ij}^{(n+1)} = g_{ij}^{(n)} + \delta g_{ij}^{(n)} \tag{35}$$

where $\delta g_{ij}^{(n)}$ is the result of a local matching transformation that minimizes the mismatch of the excitation gradients and the environment. In this case, it is the excitation that adapts and rebuilds its metric, becoming part of the environment. In this case, it is the excitation that adapts and rebuilds its metric, becoming part of the environment. Since, in the case under consideration, the metric changes through the angular component of the agreement, this process can be considered as a sequence of rotation angles ϑ_n of the spin, tending to the spin angle in the environment. Denoting the metric rotation operator at step n as $R_{ij}^{(n)}$, the result of the composition of all rotations, that is, the shell metric, can be represented as

$$g_{ij}^{(\infty)} = \left(\prod_{n=0}^{\infty} R_{ij}^{(n)}\right) \cdot g_{ij}^{(0)} \tag{36}$$

That is, it is entirely possible to replace the dynamics of the metric with the dynamics of the angle and to describe the alignment trajectory as a path on the manifold of metrics. The shell is formed as the result of a cascade of microtransformations — each of which aligns the excitation with the response of the environment. In this way, one can construct a generalization in the form of an integral rotation operator

$$\mathcal{R} = \mathcal{P}exp(\int \Omega(s)ds) \tag{37}$$

where Ω(s) is the rotation operator and $\mathcal{P}$ is the path ordering. $\mathcal{P}exp(\int \Omega(s)ds)$ is a path-ordered exponential describing the continuous accumulation of rotations. The rotation of the metric is not just a motion, but a process associated with successive transitions between states. Each small rotation requires an expenditure of energy - this is a topological contribution to the Hamiltonian. In an excited state, when the spin or field follows the rotation, the energy directly participates in the process, matching the excitation with its structural organization. The resulting energy of the final configuration can be represented as

$$H = H_0 + \Delta E_{rotation} + W[\delta\rho] \tag{38}$$

Where W[δρ] is the local density functional, $\Delta E_{rotation}$ is the energy contribution from the metric rotation. In general, the energy of a trajectory, including rotation, is defined as

$$\Delta E = \min_{\gamma:\phi_{in}\to\phi_{out}} \int_{\gamma} \mathcal{L}\big(\vartheta(s), \rho(s)\big)ds \tag{39}$$

where ϕ_{in} and ϕ_{out} are the initial and consistent with the metric of the medium forms, respectively, $\mathcal{L}$ is the rotation Lagrangian, depending on the angle ϑ and the local density ρ. This is the energy of minimal action – a consequence of the consistency requirement and is exactly equal to the energy required to form a new stable state. However, in Riemannian geometry, the Lagrangian is not the difference between the kinetic and potential energies, but the energy expended on the rotation of the local form in the metric. The action is not a long time, but along the path of metric consistency. Thus, the reformulation of the Lagrangian principle is expressed by the equation

$$\delta \int \mathcal{L}\big(\vartheta(s), \rho(s)\big) ds = 0 \tag{40}$$

the solution of which is the trajectory of the metric that minimizes the energy costs of rotation, which can be interpreted as a matching wave, a wave of geometries changing the metric, which propagates along the Riemann surface, linking the initial excitation with the response form of the medium and, as in an elastic medium in which a wave transmits a disturbance, a wave on a Riemann surface is a sequential restructuring of local symmetries. Mathematically, this is formulated as

$$\phi(r, s`) = \phi^{(0)}(r) + \sum_n \delta\phi_n(r)\, \chi_n(s) \tag{41}$$

where s is the trajectory parameter in the space of metrics, $\chi_n(s)$ are the matching wave functions that ensure the transition from the initial to the excited state with minimal energy losses. If $g_{ij}(s)$ is changing along the trajectory s, then

$$\frac{d^2 g_{ij}}{ds^2} + \omega^2_{geometry} g_{ij} = F_{ij}(s) \tag{42}$$

Here $F_{ij}(s)$ is the external excitation from the wave. Note that this equation is the equation of forced oscillations of geometries. And they are precisely what are recorded in experiments when the frequency ω_geometry enters resonance and the local metric undergoes a rotation. The entire process of rotations can be interpreted as a chain of excitations of geometries in each of which the angle δϑ

It is set by the energy minimum, realized within the current local metric. The given equation reflects the process of forced rotation of the local geometry in response to excitation. When resonance is reached between the excitation and the natural frequency of the geometry of the medium, a change in orientation occurs - the result of which is fixed as a new local metric. Thus, the process of spin rotation inside the domain shell is represented as a sequential passage through a chain of Riemann surfaces, each of which corresponds to a local energy minimum and implements a certain angle of spin rotation. The more accurately the local gradients are matched, the greater the number of such matching layers - "rotation planes" This leads to an understanding of the shell as a discrete-analog structure, each element (Riemann surface) of which carries a fixed change in angle, and the entire shell acts as a matching system of geometry transformations. It is this structure - not a wave or a particle in the usual sense - that determines how the magnetization vector smoothly rotates within the domain shell, achieving a complete or partial restructuring. Thus, the shell sets the route and dynamics of the sequential restructuring of the geometry, acting as the main agent of the interaction of the domain with the environment. If we introduce a generalized metric that depends on the angle

$$g_{ij}(s) = g_{ij}^{(0)} + \delta g_{ij}\big(\vartheta(s)\big) \tag{43}$$

and the angle itself changes according to the law associated with the excitation function

$$\frac{d\vartheta}{ds} = \alpha f(s) \tag{44}$$

where α is the matching coefficient (possibly dependent on the density of the medium), *f(s)* is the local excitation, then it becomes possible to relate the rotation angle *ϑ(s)* directly to the metric.

The magnetic domain shell in $Ni_{1-x}Zn_xFe_2O_4$ ferrite materials is a narrow zone within which the magnetization vector smoothly changes its direction from one stable magnetic state to another. It is generally accepted that the rotation type can be Néel or Bloch, depending on the orientation of the spins. Typical sizes of the magnetic domain shell are 5–10 nm, according to literature data. However, new small-angle X-ray scattering experiments and atomic force microscopy observations (conducted independently in Dubna, Russia, and in Baku) revealed the presence of magnetic $Ni_{1-x}Zn_xFe_2O_4$ particles with sizes of 2–8 nm, which casts doubt on the previously established estimates [19].

It is possible that these experiments considered ferrite particles of a significantly larger size, and the observed formations of 2–8 nm in size represent local density clusters comparable in scale to a unit cell. In this case, it would be incorrect to talk about a full-fledged magnetic domain and its shell; it is preferable to use the concept of a local density cluster.

Nevertheless, the hypothesis of the participation of the oxygen ion as a matching element between two fragments of the structure, octahedral and tetrahedral, with different spin directions, deserves attention. There is a subtle but important feature: two unfilled oxygen orbitals require the same spin orientation from the captured electrons. The critical situation that arises in this case is apparently resolved at the level of docking of the structures with oxygen. In one of these bonds, the orientation of the electron spin does not completely coincide with the required direction but contains a sufficient projection. This should manifest itself not as a smooth, but as a jump-like transition in the spin direction.

The peculiarity of the spin "rotation" mechanism here is that physical rotation may not occur at all: oxygen forms two generalized orbitals - one with the iron ion in the octahedral fragment, the other - with the iron ion in the tetrahedral one. In this case, the observed rotation inside the domain is not the result of a consistent change in the spin, but an appearance caused by the spatial rearrangement of the orbital system. Smooth change in the direction of the spin of the magnetic moment is also possible. An example is the above-

described model of the functioning of the magnetic domain shell, where the magnetization vector changes continuously within the transition zone between two stable states. Such a scenario is realized in the case of a well-coordinated rotation of the orbital basis, when the orientation of the spins adapts to the spatially varying metric of the medium. It is this mechanism that is most often associated with the classical concepts of Neel and Bloch structures in ferromagnets.

It is important to note that the transition between metrics is neither isotropic nor uniform: the deformation of the metric occurs along directions determined by the internal symmetry of the excited state and the structure of the unoccupied level density. Thus, the matching is not realized throughout the shell at once, but sequentially, through a series of nested geometric adaptations, each minimizing the energy functional at its own level of nesting.

This model opens up the possibility to consider the transfer of the excited state as a process of geometric modulation of the medium, in which a real physical quantity (e.g., spin orientation or polarization) changes not due to a field in the Euclidean sense, but as a result of an internal rotation of the metric in Riemannian geometry. This, in turn, calls for a revision of standard approaches to modeling excited states in solids, especially in materials with a high degree of local anisotropy and the presence of fields with topological structures (e.g., skyrmionic or vortex modes).

Consequently, within the framework of this model, one can assert: the orientation of the spin is a mapping of a trajectory on a sphere in the coordinates of the shell, not the actual motion of the vector $\vec{S}$in Euclidean space. The rotation occurs in geometry, not in physics — and only then becomes observable as a physical effect.

A theoretical model is proposed based on a Hamiltonian for spatially inhomogeneous systems interacting with an active medium possessing internal resonant properties. In this framework, both kinetic and potential energies depend on the local geometry and the excitation field, enabling the description of the energy structure of excited states, including spin waves, solitons, and stable topological configurations. The model accounts for the self-consistent influence of the medium and its hidden symmetry, going beyond conventional theoretical potentials. The proposed representation proves especially effective in interpreting theoretical and experimental studies of materials with nonlinear responses and spin structures, such as ferrite films, in which anisotropy and geometrically induced effects play a key functional role.

A number of publications provide various estimates of the parameters of the structural and dynamic characteristics of $Ni_{1-x}Zn_xFe_2O_4$ ferrites.

In this paper, to interpret the experimental data, we use parameters consistent with the results of independent measurements performed in leading research centers, as well as with our own series of experiments on thin films produced by laser ablation. It should be noted that some of the previously published data (e.g., concerning the placement of Zn^{2+} in octahedral positions or Fe^{2+} concentrations under normal film production conditions) are not confirmed by modern high-precision measurements.

In the center of the domain, where high-frequency precession modes for ferrites of the $Ni_{1-x}Zn_xFe_2O_4$ type predominate, the characteristic frequency is estimated by the formula:

$$f_{core} = \frac{\gamma}{2\pi} H_{eff} \approx 5\text{–}15 \text{ GHz}$$

with an effective field of the domain core $H_{eff} \sim 1000$–2000 Oe.

In the domain shell, the effective field decreases sharply (to 10–20% of H_{eff}), due to a decrease in exchange interaction, weakening of anisotropy and, accordingly, low-frequency relaxation modes arise.

The characteristic frequencies of the magnetic domain shell oscillations were estimated based on the shell model as a transition layer with a reduced H_{eff}, determined by residual anisotropy and weakened exchange interaction. Taking:

$$H_{eff}^{shell} \approx 50\text{–}200 \text{ Oe}$$

pre-session frequencies of shell spin oscillations are estimated as:

$$f_{core} = \frac{\gamma}{2\pi} H_{eff}^{shell} \approx 20\text{-}100 \text{MHz}$$

where $\gamma \approx 2.8$ MHz/Oe is the gyromagnetic ratio for $Ni_{1-x}Zn_xFe_2O_4$ ferrites.

This estimate is in good agreement with the experimental results reported in the literature [17], where the peak frequencies of domain wall dynamics are ~3.5–20 MHz. It is in this region, as shown by Mössbauer spectroscopy data (relaxation line width corresponding to times of the order of $10^{-9} – 10^{-8}$ s), that the domain shell “lives”. On the other hand, SANS experiments allow us to estimate the presence of shell vibrational modes in the range of hundreds of megahertz.

In the constructed model of the domain shell based on nested Riemann sheets, the minimum thickness of one sheet is determined by the structural unit of the Fe–O–Fe superexchange interaction, which sets the lower limit of the shell thickness at the level of $\delta d \geq 3$ Å. For domains with a characteristic size $D \sim 10$ nm, the shell thickness is about 5% of the domain size, which corresponds to the nesting of two Riemann sheets providing a coordinated spin rotation of 180° (90° per sheet).

The key assumption of the model is the synchronization of all observed vibrational modes of the shell with the fundamental modes of the domain: each stable mode of the shell is an overtone, i.e. a term of the expansion in the fundamental modes of the domain. Such coordination of phases and frequencies explains the mode structure observed in several spectral experiments (in particular, the mode ~880 cm^{-1}), corresponding to the overtone nature of the shell.

Since the Raman spectrum shows a stable mode at ~880 cm^{-1} (~26.39 THz), which depends on the composition and thickness of the ferrite film, we should expect the formation of structured hybrid modes in the range of 0.1–1 THz due to the spin-phonon interaction in the domain shell. Dynamic deformations of the shell caused by the transition between the ordered internal region and the external environment lead to the appearance of a THz structure reflecting a complex modulation of the local lattice.

A pronounced spectral structure in the range of 1.2–15 THz was experimentally recorded in the studied $Ni_{1-x}Zn_xFe_2O_4$ films. The observed modes are interpreted as the result of spin-phonon interaction in the domain shell, where frequencies of 1–3 THz correspond to hybrid acoustomagnetic modes of the transition layer, and the high-frequency component (5–15 THz) is due to residual optical phonons of tetrahedral bonds modulated by the shell dynamics.

Considering the limitations of the instrumental sensitivity in the low-frequency region (30–50 cm^{-1}), it can be considered that the available data sufficiently confirm the overtone structure of the domain shell and allow us to interpret the observed effects within the framework of the proposed model.

The observation of the coincidence of the ~880 cm^{-1} mode in the two-photon luminescence spectra and the THz reflection and transmission spectra in independent experiments performed on $Ni_{1-x}Zn_xFe_2O_4$ films created by laser ablation by employees of the Institute of Physics of the National Academy of Sciences of Belarus (using powder materials obtained at the Institute of Physics of the National Academy of Sciences of Azerbaijan) confirms the non-artifactual nature of this mode and its belonging to the dynamic structure of the shell.

In addition, the presence of an ordered sequence of maxima in the THz spectra indicates the formation of a hybrid lattice in the domain shell. It should also be noted that the Mössbauer studies agree well with the presented conclusions ($\tau_{relax} \sim 10^{-12}$ s), which correspond to the proposed model both in terms of the characteristic frequencies and in terms of the dynamic behavior of the cladding modes [24-28]. The table shows a comparison of the frequencies calculated within the model with the results of published studies:

Source	Frequency
Proposed model H_{eff}^{shell}	~20–100 MHz
Hwang J. et al. [16]	~20 MHz
Stojanović et al. [17]	~5–16 MHz
Tsutaoka T. [22]	~3.5 MHz

CONCLUSION

The approaches presented in this work offer a new perspective on the structure of the Hamiltonian—as a functional determined not only by the local wavefunction, but also by the unfixed geometry of the medium. In this formulation, the metric generates a spatial modulation of the internal properties of the medium and depends, for instance, on the distribution of interaction forces—such as anisotropic exchange, defects, and local symmetries.

This enables the consideration of multilayered interaction physics in materials with anisotropy, chirality, and excitation gradients. Such a formulation allows modeling not only stable configurations (vortices, skyrmions, domains), but also transitional states arising from symmetry breaking or external influence.

The apparent simplicity of interpretation in the proposed model stems from the use of Riemann surfaces, whose intrinsic properties determine the observed transformations. These transformations cannot be explained within Euclidean geometry due to its integrative nature and the loss of local structural detail.

A key insight is the representation of Riemann surfaces as the foundation of internal states of the medium: all homogeneous objects are placed on a single Riemann surface, and it is through these surfaces that permissible transformations are described. This is especially promising for the analysis of coherent quantum objects, including nonlinear excitations, topological defects, and ordering dynamics in active media.

The analysis of the magnetic domain shell mechanism, exemplified by $Ni_{1-x}Zn_xFe_2O_4$ ferrites, has demonstrated the importance of a coherent geometric description of spin rotation. In this context, orientation can change without physical rotation—through the restructuring of coupled orbitals.

Such behavior is characteristic of antisymmetric objects, where a change in orientation produces an opposite physical effect (e.g., a reversed spin); of phase transitions, where the new phase emerges not via surface disruption but through a shift into a different region of parameter space; of interlayer couplings, where orientation matters more than absolute form (as in valence interactions or orientational domains); and of magnetic shells, where spin direction changes while the geometry of the boundary remains constant.

This phenomenon is what we call the Euclidean illusion of transformation: a change appears to occur, but there is only a transition to another allowed orientation on the same Riemann surface. Mathematically, this is not a change of shape but a shift within the symmetry group.

While Euclidean geometry provides an integral picture, it obscures the internal structure of such transitions—granting interpretive freedom to the observer but distancing them from the true mechanism. The model proposed here resolves this contradiction, enabling a precise description of physical transformations using the tools of differential geometry and phase-structured media.

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