DETERMINATION OF THE TEMPERATURE DEPENDENCE OF THE HEAT CAPACITY OF LIQUIDS BY THERMOHYDRODYNAMIC METHOD

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The determination of the temperature dependence of the heat capacity of liquids based on the thermohydrodynamic method was studied in this article theoretically and mathematically. The problem of determining the temperature dependence of the isobaric heat capacity in the study was reduced to the solution of the inverse problem for a nonlinear special derivative differential equation. In order to reduce the complexity of the analytical solution, the mathematical model was simplified and approximate solution methods were developed for special cases, taking into account the effects of various external physical fields - a constant electric field and acoustic waves. Formulas were obtained for determining the nonlinearity coefficient characterizing the dependence of the heat capacity on temperature by applying the “fork” method under conditions of monotonic heating of a liquid in an unsteady temperature field. The results show that as the intensity of external physical fields increases, the nonlinearity criterion increases proportionally. The proposed approach creates a theoretical basis for controlling and optimizing processes in the oil, chemical and thermal power industries.

Keywords: liquid, heat capacity, temperature, electric field, acoustic wave.
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INTRODUCTION

Thermal physical parameters of liquids are quantities that reflect the internal structure of the liquid, the processes occurring inside the liquid, and are the first to react to external influences. Since it is possible to change the thermal physical parameters of liquids under the influence of external physical fields and adjust them in the necessary direction, it has been proposed in recent years to use the influence of physical fields to increase the efficiency of certain technological processes in the oil, chemical, and thermal power industries [1-4]. Various processes occur during the interaction of external physical fields with the liquid. The nature and significance of the resulting processes depend on the intensity of the external physical field. The absorbed energy of the external physical field leads to an increase in the internal energy of the liquid, which in turn leads to a change in the thermal physical parameters of the liquid. In other words, the thermal physical parameters of the liquid change depending on the temperature. This happens when the intensity of the external physical field is weak. When the intensity of the external physical field is sufficiently large, a structural change occurs inside the liquid during the interaction process. Thus, the regulation of the thermal physical parameters of the liquid by temperature can be considered the most effective method. For this reason, the study of the change in the thermal physical parameters of liquids depending on temperature has been the focus of researchers for many years. One of the most important thermal physical parameters of liquids is their heat capacity. The temperature dependence of the heat capacity of liquids has been studied mainly for a steady temperature field. Such studies have two main shortcomings: 1) the fact of changing of all real technological processes depending of the temperature field on time, i.e., disregarding their unsteadiness; 2) high time-consuming of experiments in laboratory conditions, i.e., taking a long time of the processes to reach a steady state.

EXPERIMENTAL PART AND DISCUSSION OF RESULTS

Analytical study of the temperature dependence of isobaric heat capacity is mathematically reduced to the solution of the inverse problem for the heat transfer equation. In this case, since the differential equation is nonlinear, finding its exact analytical solution poses serious mathematical difficulties. The study of the temperature dependence of the heat capacity of liquids by the thermohydrodynamic method is reduced to the solution of the following differential equation for a one-dimensional temperature field [5]:

$$C_p(T)\rho(T)\left(\frac{\partial T}{\partial t}+\upsilon\frac{\partial T}{\partial x}\right)=\frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right]+Q(T), \tag{1}$$

where $Q(T)$ - is the volume density of heat received from the outside.

First, let us define some particular solutions for the steady-state temperature field case. To find an effective solution to the differential equation (1), let us consider some simplifications in this mathematical model. Let us assume that the fluid moves in a steady-state temperature field and interacts with an acoustic wave meanwhile. The volume density of the energy absorbed by the acoustic wave in the fluid is:

$$Q(x)=\alpha(x)J_0\exp\left(\int_0^x \alpha(x)dx\right), \tag{2}$$

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where $\alpha(x)$ - is the absorption coefficient of the acoustic wave, J_0 - is the intensity of the acoustic wave.

Simple calculations show that the amount of heat transferred by convection as the liquid moves through the tube is greater than the amount of heat transferred by diffusion. This condition is mathematically written as follows [6]:

$$C_p(T)\rho(T)\upsilon\frac{\partial T}{\partial x} >> \frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right]. \quad (3)$$

Taking into account the above, we can write the differential equation (1) as follows:

$$\int_{T_1}^{T_2} C_p(T)\rho(T)dT = \int_0^x \frac{\alpha(x)}{\upsilon} J_0\left[\exp\int_0^x \alpha(x)dx\right]dx. \quad (4)$$

The determination of the temperature dependence of the heat capacity of liquids is reduced to the solution of the integral equation (4). The solution of equations of this type is usually sought in the form of a polynomial series. For clarity, let us assume that:

$$C_p(T)\rho(T) = C_p(0)\rho(0)\exp[k(T-T_0)]. \quad (5)$$

Thus, the determination of the temperature dependence of the heat capacity of the liquid is reduced to the solution of the constant "k". If we substitute expression (5) in equation (4) and integrate it, and if we separate the expressions $exp[k(T_2-T_0)]$, $exp[k(T_1-T_0)]$into a series and settle for the first three terms of the series, we get the following formula for the constant "k":

$$k = \frac{2}{2T_0-(T_1+T_0)}\left[1+\frac{J_0}{C_p(0)\rho(0)\upsilon\Delta T}\left[\exp\left(\int_0^l \alpha(x)dx\right)-1\right]\right]. \quad (6)$$

It is clear from the above-mentioned that even in the simplest case, determining the temperature dependence of the heat capacity of a liquid is reduced to solving an integral equation. The "fork" method is used to simplify the solution of the problem. Applying this method, let us determine the temperature dependence of the heat capacity of a liquid. For this purpose, let us use a simplified mathematical model. For clarity, let us assume that:

$$C_p(T)\rho(T) = C_p(0)\rho(0)(1+kT). \quad (7)$$

The fluid flowing in the pipe is an electrical conductor and since it is under the influence of a constant electric field, absorbed energy density of the field is $Q=\sigma E^2$. Under the mentioned conditions, the temperature dependence of the heat capacity is reduced to the solution of the following equation. In the "Fork" method, equation (7) is solved for two cases: 1) $T=T_{max}$ and 2) $T=T_{min}$, that is, it is written as follows:

$$C_p(0)\rho(0)(1+kT_{\max})\upsilon\frac{dT}{dx} = \sigma E^2$$

and (8)

$$C_p(0)\rho(0)(1+kT_{\min})\upsilon\frac{dT}{dx} = \sigma E^2.$$

These equations are solved and k_{max} and k_{min}were determined. Their average value, i.e.

$$k_{or} = \frac{k_{\max}+k_{\min}}{2}. \quad (9)$$

It is the average value of the nonlinearity coefficient characterizing the dependence of the heat capacity of the liquid on temperature. We get following from equations (8):

$$k_{\max} = \frac{1}{T_{\max}}\left[\frac{\sigma E^2 l}{C_p(0)\rho(0)\upsilon\Delta T_{\max}}-1\right], \Delta T_{\max} = T_{1\max}-T_{2\max} \quad (10)$$

$$k_{\min} = \frac{1}{T_{\min}}\left[\frac{\sigma E^2 l}{C_p(0)\rho(0)\upsilon\Delta T_{\min}}-1\right], \Delta T_{\min} = T_{1\min}-T_{2\min}. \quad (11)$$

Let us determine the temperature dependence of the heat capacity of a liquid in a given temperature range using the simplified mathematical model mentioned above. In monotonic heating of a liquid $\frac{\partial T}{\partial t} = A = const.$ $T(t) = At + B$. It should be noted that in practice, realizing monotonic heating of a liquid is a very difficult problem. Taking into account the above, we can write the differential equation (1) as follows:

$$C_p(T)\left(A+\upsilon\frac{dT}{dx}\right)=\sigma E^2 \qquad (12)$$

For the clarity, let us assume that the dependence of isobaric heat capacity on temperature is determined by formula (7). Using the "fork" method, let us determine the constant quantity "*k*". We formulate equation (12) for the maximum and minimum values of temperature:

$$C_p(0)\rho(0)(1+kT_{\max})\left(A+\upsilon\frac{dT}{dx}\right)=\sigma E^2 \qquad (13)$$

and

$$C_p(0)\rho(0)(1+kT_{\min})\left(A+\upsilon\frac{dT}{dx}\right)=\sigma E^2. \qquad (14)$$

These equations have been solved under the following conditions:

$$T(0)=T_{1\max}, \quad T(l)=T_{2\max}, \quad \Delta T_{\max}=T_{2\max}-T_{1\max}$$
$$T(0)=T_{1\min}, \quad T(l)=T_{2\min}, \quad \Delta T_{\min}=T_{2\min}-T_{1\min}.$$

We obtain the following expressions for the maximum nonlinearity criteria of "*k*" and minimum nonlinearity criteria of "*k*":

$$k_{\max}=\frac{1}{T_{\max}}\left[\frac{\sigma E^2}{C_p(0)\rho(0)\left(A+\frac{\upsilon}{l}\Delta T_{\max}\right)}-1\right],$$

$$k_{\min}=\frac{1}{T_{\min}}\left[\frac{\sigma E^2}{C_p(0)\rho(0)\left(A+\frac{\upsilon}{l}\Delta T_{\min}\right)}-1\right].$$

The average value of the nonlinearity criterion constant "*k*" is found according to formula (9).

CONCLUSION

The following conclusions can be made based on the analytical calculations:

1. The analytical determination of the temperature dependence of the heat capacity of liquids is mathematically reduced to the solution of the inverse problem for a nonlinear second-order differential equation with a particular derivative.
2. Since the exact analytical solution of the nonlinear differential equation is associated with serious mathematical difficulties, approximate cases were constructed for several special cases by simplifying the mathematical model.
3. An approximate solution was found under the condition of monotonous heating of the liquid in an unsteady temperature field, and a formula for the temperature dependence of the heat capacity of the liquid was proposed.
4. The effect of a constant electric field and acoustic waves on the temperature dependence of the heat capacity of liquids was analytically studied. According to the obtained formulas, the nonlinearity criterion increases as the intensity of external physical fields increases.

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