QUANTIZATION OF THE ELECTRIC CHARGE OF LEPTONS

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The possibility of obtaining quantization of the electric charge of leptons in the Weinberg-Salam model by introducing the right-hand component of the electron into the isodoublet is investigated. However, the relations arising from the anomaly cancellation conditions are not used. It is shown that the interaction of the Higgs fields with lepton isomultiplets leads to certain relations between the hypercharges of these fields. This indicates the influence of the Higgs field on the "formation" and quantization of electric charge.

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A number of papers [1-5] was dedicated to the study of the quantization of electric charge in the Standard Model (SM) and its extensions. In [6-8], we investigated the quantization of the electric charge of leptons and quarks in the SM. Unlike [1-5], in [6-8] did not use relations that follow from the anomaly cancellation conditions. In [6, 8], the right-hand component of the electron is introduced as an isosinglet e_R^-. In this paper, we introduce the right-hand component of the electron as a component of isodoublet $\begin{pmatrix} E^0 \\ e^- \end{pmatrix}_R$ (E^0 − a new lepton). As in [6-8], here either we will not use relations that follow from the anomaly cancellation conditions.

Thus, in this case we have the following lepton fields:

$$\psi_{eL} = \begin{pmatrix} \nu_e(\alpha) \\ e^- \end{pmatrix}_L , \quad \psi_{eR} = \begin{pmatrix} E^0 \\ e^- \end{pmatrix}_R , \quad \psi'_L = E_L^0(\alpha) \tag{1}$$

and the Higgs isodoublet φ and isosinglet ϕ :

$$\varphi = \begin{pmatrix} \varphi_+ \\ \varphi_0 \end{pmatrix}, \ \phi; \tag{2}$$

where

$$\nu_{eL}(\alpha) = \nu_{eL} cos\alpha + E_L^0 sin\alpha ,$$
$$E_L^0(\alpha) = E_L^0 cos\alpha - \nu_{eL} sin\alpha ,$$

(we introduce mixing ν_{eL} and E_L^0 to eliminate off-diagonal terms from the mass part of the Lagrangian). The Lagrangian for the interaction of leptons and Higgs fields with gauge fields is obtained by replacing

$$\partial_\mu \to D_\mu = \partial_\mu - ig\left(\vec{t}\vec{b}_\mu\right) - i\frac{Y}{2} g_1 a_\mu \tag{3}$$

in the Lagrangian of the Higgs field and the free massless Dirac field $i\overline{\psi}\gamma_\mu\partial_\mu\psi$, and the desired Lagrangian has the form

$$L = i\overline{\psi}_{eL}\gamma_\mu D_\mu^L \psi_{eL} + i\overline{\psi}_{eR}\gamma_\mu D_\mu^R \psi_{eR} + i\psi'_L\gamma_\mu D_\mu^{L'}\psi'_L + \left(D_\mu^\varphi \varphi\right)^+ \left(D_\mu^\varphi \varphi\right) + \left(D_\mu^\phi \phi\right)^+ \left(D_\mu^\phi \phi\right) \tag{4}$$

Here $g, g_1 -$ are the interaction constants, $\vec{t}$ and $Y -$ is the isospin and hypercharge operator of a given multiplet, $\vec{b}_\mu -$ is the Yang-Mills isotriplet, $a_\mu -$ is the Maxwell isosinglet; ψ_{eL}, ψ_{eR} and $\psi'_L -$ are the left and right isomultiplets of the lepton fields. We do not accept the Gell-Mann–Nishijima relation for weak hypercharges. We will accept them as free parameters and will try to fix them based on physical requirements.

We denote the hypercharges of the lepton and Higgs fields as follows

$$\mathrm{Y}(\psi_{eL}) = y_L , \mathrm{Y}(\psi_{eR}) = y_R , \mathrm{Y}(\psi'_L) = y_1 , \mathrm{Y}(\varphi) = y_\varphi , \mathrm{Y}(\phi) = y_\phi \tag{5}$$

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and assume they are real.

As is well known, for isoscalar fields $E_L^{'}$ and ϕ: $\vec{t}=0$, and for isodoublets ψ_{eL},ψ_{eR} and φ: $\vec{t}=\vec{\tau}/2$ ($\vec{\tau}-$ Pauli matrices). We write the transformation of the fields b_μ^3 and a_μ into physical fields A_μ and Z_μ as

$$b_\mu^3 = A_\mu \sin\theta + Z_\mu \cos\theta,$$
$$a_\mu = A_\mu \cos\theta - Z_\mu \sin\theta, \qquad (6)$$

where $\sin\theta = \frac{g_1}{\sqrt{g^2+g_1^2}}$, $\cos\theta = \frac{g}{\sqrt{g^2+g_1^2}}$

Taking into account (5) in (4), for the interaction of leptons with gauge fields, we have

$$L_l = \overline{\psi}_{eL}(i\gamma_\mu D_\mu^L)\psi_{eL} + \overline{\psi}_{eR}(i\gamma_\mu D_\mu^R)\psi_{eR} + \overline{\psi}_L^{'}(i\gamma_\mu D_\mu^{L'})\psi_L^{'} =$$
$$= \overline{\psi}_{eL}\gamma_\mu[i\partial_\mu + \frac{1}{2}(g\vec{\tau}\vec{b}_\mu + g_1 y_L a_\mu)]\psi_{eL} + \overline{\psi}_{eR}\gamma_\mu\left[i\partial_\mu + \frac{1}{2}(g\vec{\tau}\vec{b}_\mu + g_1 y_R a_\mu)\right]\psi_{eR} + \qquad (7)$$
$$+ \overline{\psi}_L^{'}\gamma_\mu(i\partial_\mu + \frac{1}{2}g_1 y_1 a_\mu)\psi_L^{'} \equiv L_{KIN} + L_{CC} + L_{NC}.$$

Here $L_{KIN} = i\overline{\nu}_{eL}(\alpha)\hat{\partial}\nu_{eL}(\alpha) + i\overline{e}_L^-\hat{\partial}e_L^- + i\overline{e}_R^-\hat{\partial}e_R^- + i\overline{E}_R^0\hat{\partial}E_R^0 + i\overline{E}_L^0(\alpha)\hat{\partial}E_L^0(\alpha) -$
is the kinetic part of the Lagrangian,

$$L_{CC} = \frac{g}{\sqrt{2}}\left[\left(\overline{\nu}_{eL}(\alpha)\gamma_\mu e_L^- + \overline{E}_R^0\gamma_\mu e_R^-\right)W_\mu^+ + \left(\overline{e}_L^-\gamma_\mu\nu_{eL}(\alpha) + \overline{e}_R^-\gamma_\mu E_R^0\right)W_\mu^-\right] -$$

is the Lagrangian of the interaction of charged currents (CC),

$$L_{NC} = \frac{1}{2}\overline{\nu}_{eL}(\alpha)\gamma_\mu\nu_{eL}(\alpha)(gb_\mu^3 + g_1 y_L a_\mu) + \frac{1}{2}\overline{e}_L\gamma_\mu e_L(-gb_\mu^3 + g_1 y_L a_\mu) + \frac{1}{2}\overline{e}_R\gamma_\mu e_R(-gb_\mu^3 + g_1 y_R a_\mu) +$$
$$+ \frac{1}{2}\overline{E}_R^0\gamma_\mu E_R^0(gb_\mu^3 + g_1 y_R a_\mu) + \frac{1}{2}\overline{E}_L^0(\alpha)\gamma_\mu E_L^0(\alpha)g_1 y_1 a_\mu -$$

is the Lagrangian of the interaction of neutral currents (NC).

Taking into account the relations $\bar{f}_L\gamma_\mu\psi_L = \frac{1}{2}\bar{f}O_\mu\psi$, $\bar{f}_R\gamma_\mu\psi_R = \frac{1}{2}\bar{f}O_\mu^{'}\psi$ and moving on to the physical fields A_μ and Z_μ (relationship (6)), for the parts of the Lagrangians L_{CC} and L_{NC} , we have the following expressions:

$$L_{CC} = \frac{g}{2\sqrt{2}}\left[(\overline{\nu}_{eL}(\alpha)O_\mu e^- + \overline{E}^0 O_\mu^{'} e^-)W_\mu^+ + (\overline{e}^- O_\mu \nu_e(\alpha) + \overline{e}^- O_\mu^{'} E^0)W_\mu^-\right] \qquad (8)$$

$$L_{NC} = \frac{1}{4}\overline{\nu}_e(\alpha)O_\mu\nu_e(\alpha)(gb_\mu^3 + g_1 y_L a_\mu) + \frac{1}{4}\overline{e}^-\gamma_\mu\left[-2gb_\mu^3 + g_1(y_L + y_R)a_\mu + \gamma_5 g_1(y_L - y_R)a_\mu\right]e^- +$$
$$+ \frac{1}{4}\overline{E}^0\gamma_\mu\left[gb_\mu^3 + g_1(y_1 + y_R)a_\mu + \gamma_5\left(-gb_\mu^3 + g_1(y_1 - y_R)a_\mu\right)\right]E^0 = \overline{\nu}_e(\alpha)\gamma_\mu\left(Q_\nu + Q_\nu^{'}\gamma_5\right)\nu_e(\alpha)A_\mu + \overline{e}^-\gamma_\mu\left(Q_{oe} + Q_{oe}^{'}\gamma_5\right)e^- A_\mu +$$
$$+ \overline{E}^0\gamma_\mu\left(Q_{E^0} + Q_{E^0}^{'}\gamma_5\right)E^0 A_\mu + \overline{\nu}_e(\alpha)\gamma_\mu\left(g_V^\nu + g_A^\nu\gamma_5\right)\nu_e(\alpha)Z_\mu + \overline{e}^-\gamma_\mu\left(G_V^e + G_A^e\gamma_5\right)e^- Z_\mu + \overline{E}^0\gamma_\mu\left(G_V^{E^0} + G_A^{E^0}\gamma_5\right)E^0 Z_\mu$$

were

$$O_\mu = \gamma_\mu(1+\gamma_5),\quad O_\mu^{'} = \gamma_\mu(1-\gamma_5),\quad W_\mu^{\pm} = \frac{1}{\sqrt{2}}(b_\mu^1 \mp ib_\mu^2),\quad \hat{\partial} = \gamma_\mu\partial_\mu,\ \overline{\psi} = \psi^+\gamma_4,$$

$$Q_\nu = \frac{1}{4}\left[g\sin\theta + g_1 y_L\cos\theta\right],\qquad Q_{0e}^{'} = \frac{1}{4}g_1(y_L - y_R)\cos\theta,$$
$$Q_\nu^{'} = \frac{1}{4}\left[g\sin\theta + g_1 y_L\cos\theta\right],\qquad Q_{E^0} = \frac{1}{4}\left[g\sin\theta + g_1(y_1 + y_R)\cos\theta\right], \qquad (9a)$$

$$Q_{0e} = \frac{1}{4}\left[-2g\sin\theta + g_1(y_L + y_R)\cos\theta\right], \qquad Q'_{E^0} = \frac{1}{4}\left[-g\sin\theta + g_1(y_1 - y_R)\cos\theta\right],$$

$$g_V^{\nu} = \frac{1}{4}\left[g\cos\theta - g_1 y_L \sin\theta\right], \qquad G_A^{e} = \frac{1}{4} g_1(y_R - y_L)\sin\theta,$$
$$g_A^{\nu} = \frac{1}{4}\left[g\cos\theta - g_1 y_L \sin\theta\right], \qquad G_V^{E^0} = \frac{1}{4}\left[g\cos\theta - g_1(y_1 + y_R)\sin\theta\right], \tag{9b}$$
$$G_V^{e} = \frac{1}{4}\left[-2g\cos\theta - g_1(y_L + y_R)\sin\theta\right], \qquad G_A^{E^0} = \frac{1}{4}\left[-g\cos\theta - g_1(y_L - y_R)\sin\theta\right].$$

From expressions (9a) it is clear that, as in the case of work [6], here too the P-invariance of the electromagnetic interaction and the electroneutrality condition for the neutrino are equivalent.

In the case under consideration, the lepton masses are generated using the Yukawa mass Lagrangian

$$L^{l}_{mass} = \frac{m_e}{\langle\phi\rangle}\overline{\psi}_R\psi_L\phi + \frac{m_{E^0}\cos\alpha}{\langle\varphi_0\rangle}\overline{\psi}_R\psi'_L\varphi^c + h.c. \tag{10}$$

where $\varphi^c = i\tau_2\varphi^*$, $\sin\alpha = \dfrac{m_e}{m_{E^0}}$.

Taking into account the conservation of hypercharge, from (10) we obtain

$$y_R = y_L + y_\phi,\; y_R = y_1 - y_\varphi. \tag{11}$$

Taking into account (11) in (9a) we obtain

$$Q'_{E^0} = \frac{1}{4}\left(-g\sin\theta + g_1 y_\varphi \cos\theta\right) \tag{12}$$

If we take into account the P – invariance of the electromagnetic interaction of the new lepton E^0 ($Q'_{E^0} = 0$) and the electron ($Q'_{0e} = 0$), from (9a) and (12) we obtain

$$y_L = y_R,\; g_1 y_\varphi \cos\theta = g\sin\theta. \tag{13}$$

Taking into account these relations and (11) in expressions (9a), we obtain

$$Q'_\nu = Q_\nu = \frac{Q_e}{4}\left(1 + \frac{y_L}{y_\varphi}\right), \qquad Q_{0e} = -\frac{Q_e}{2}\left(1 - \frac{y_L}{y_\varphi}\right), \qquad Q_{E^0} = \frac{Q_e}{2}\left(1 + \frac{y_L}{y_\varphi}\right) \tag{14}$$

where $Q_e = g\sin\theta = |e|$ is the absolute value of the electron charge.

From (14), it is clear that the electric charges of the leptons depend on the hypercharge of the Higgs field, and this can be seen as evidence of quantization of the electric charge of the leptons. If we take into account the P-invariance of the electromagnetic interaction of neutrinos ($Q'_\nu = 0$) from (14), (13), and (11), we obtain

$$y_L = y_R = -y_\varphi,\; y_\phi = 0. \tag{15}$$

If in (9b), we take into account relations (11), (13), and (15), then for the vector and axial constants of the interactions of leptons with the Z boson, we obtain

$$g_V^{\nu} = g_A^{\nu} = \frac{g}{4\cos\theta}, \qquad G_V^{e} = \frac{g}{4\cos\theta}\left(-2 + 4\sin^2\theta\right), \qquad G_A^{e} = 0,$$
$$G_V^{E^0} = -G_A^{E^0} = \frac{g}{4\cos\theta}. \tag{16}$$

From expression (16) it is evident that the constants of interactions of leptons with the Z boson coincide with the expressions in the work [9] and when obtaining them for the hypercharge of particles, specific numbers were not taken.

Taking into account the covariant derivatives D_μ^φ , D_μ^ϕ from (4) and moving on to the vacuum expectation values of the fields (2) $\langle\varphi\rangle = \begin{pmatrix} 0 \\ \langle\varphi_0\rangle \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \eta \end{pmatrix}$, $\langle\phi\rangle = \frac{\upsilon}{\sqrt{2}}$

we obtain the following expression for the part of the Lagrangian responsible for the masses of the vector bosons:

$$\left(D_\mu^\varphi\varphi\right)^+\left(D_\mu^\varphi\varphi\right)+\left(D_\mu^\phi\phi\right)^+\left(D_\mu\phi\right)=M_W^2W_\mu^+W_\mu^-+\frac{1}{2}M_Z^2Z_\mu^2+\frac{1}{2}M_A^2A_\mu^2+M_ZM_AA_\mu Z_\mu \tag{17}$$

where

$$M_W^2=\frac{g^2\langle\varphi_0\rangle^2}{2},\ M_Z^2=\frac{1}{2}\left[\langle\varphi_0\rangle^2\left(g\cos\theta+g_1y_\varphi\sin\theta\right)^2+\langle\phi\rangle^2\left(g_1y_\phi\sin\theta\right)^2\right],$$

$$M_A^2=\frac{1}{2}\left[\langle\varphi_0\rangle^2\left(g\sin\theta-g_1y_\varphi\cos\theta\right)^2+\langle\phi\rangle^2\left(g_1y_\phi\cos\theta\right)^2\right]$$

the mass terms of the fields $W_\mu^\pm$, Z_μ and A_μ.

If we take into account relations (13) and (15) in expressions (17), then for the mass of the vector fields we have

$$M_W=\frac{g\langle\varphi_0\rangle}{\sqrt{2}},\quad M_A=0,\quad M_Z=\frac{M_W}{\cos\theta}\ .$$

Thus, from the P-invariance of the electromagnetic interactions of the neutrino $\left(Q_\nu^{'}=0\right)$, electron $\left(Q_{oe}^{'}=0\right)$ and E^0 lepton $\left(Q_{E^0}^{'}=0\right)$ it follows that the photon mass is zero, and between the masses of the $W_\mu^\pm$ and Z - bosons there is a well-known relation $M_W=M_Z\cos\theta$.

Considering relation (15) in expressions (14), for the charge of the leptons we have

$$Q_\nu=0\,,Q_{E^0}=0\,,Q_{0e}=-Q_e=-|e|\,. \tag{18}$$

In conclusion, we note that the dependence of the electric charge of leptons on the hypercharge of the Higgs field (expression (14)) and the fixation of the hypercharge of lepton isodoublets by the hypercharge of the Higgs field (relation $y_L=y_R=-y_\varphi$) can be interpreted as the influence of the Higgs field on the quantization of the electric charge of leptons. This means that it can be assumed that Higgs fields, interacting with particles, give them mass and also play a role in the "formation" of electric charge.

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